IMAGE ANALYSIS USING AUTOMATA

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Abstract. The mathematical apparatus of continued logic in the problem of the complex images analysis is given. The effective solution of this problem can be get by using of the automaton model.

1. The mathematical apparatus of continued logic is given in the work [1] and the simplest cases of its application, connected with informatics: geometric modeling, taking of the approximate solutions, optimization are described. There are many other problems from the field of informatics, which are effectively sold with the help of continued logic apparatus and its generalizations. In particular, when the object of research is complex rather effective is application of the logic determinants. At the research of the complex objects the "naive" modeling of the simple objects [1] in the continued logic terms is not applicable and we should seek the special models for getting the adequate continued-logical object description. The most resultative model of such kind is the finite dynamical automaton [2–4].

In this article we speak about images analysis problem, which could be solved by using automaton model. The essence of this problem is: we have an image in the form of collection of the visual objects in the space of given dimension and by formal way to calculate characteristics of the interlocated objects for full understanding.

2. Let's consider random N-face Cartesian space E^N filled with N-face objects with general number of objects n. The objects are considered to be intersectional: filling of any object of any field $E_i^N \in E^N$ does not prevent lying in E_i^N of the other object. Position of every object in the space E^N is fully defined. It is evidently that image P which is the result of all E^N space objects interaction we can give in such form:

$$P = P_0 \cup P_1 \cup \mathrm{K} \cup P_n, \tag{1}$$

where $P_k (k = \overline{0,n})$ – image of k different objects intersection, including images $P(i_1, i_2, ..., i_k)$ of all possible combinations $(i_1, i_2, ..., i_k)$ of k intersection objects

$$P_k = \bigcup_{i_1 < i_2 < ... < i_k} P_k(i_1, i_2, ..., i_k), k = \overline{1,n}, \tag{2}$$

where P_0 – image in some field E_0^N of the space E^N that contains no one object; P_1 – image in the other field E_1^N of the space E^N, which has only single non-intersecting with others objects; P_2 – image in the third field E_2^N of the space E^N, in which we have only pairwise intersecting objects and so on and P_n – image in the field E_n^N of the space E^N, containing only intersection of all n objects.

It's clearly that

$$E^N = E_0^N \cup E_1^N \cup \mathrm{K} \cup E_n^N; \; E_i^N \cap E_j^N = \varnothing, \; i \neq j. \tag{3}$$

In its turn

$$E_k^N = \bigcup_{i_1 < i_2 < ... < i_k} E_k^N (i_1, i_2, ..., i_k), \;\; E_k^N (I) \cap E_k^N (J) = \varnothing, \;\; I \neq J, \tag{4}$$

where $E_k^N(J)$ – the field of space E^N, where is intersection only of the concrete set k of the objects $J=(i_1,...,i_k)$.

The Problem 1 is to find in the field E_k^N according to images P_k in the form of any fixed number' intersections k non-concrete objects $(k=\overline{0,n})$. The problem 2 is to find fields $E_k^N(J)$ according to images $P_k(J)$ in the form of intersections $J=(i_i,...,i_k)$ of any number k concrete objects $i_1,...,i_k$ $(k=\overline{1,n})$. The problem 2 is more detailed. From its solution we always can get the solution of the problem 1, using formula (4). Let's call the problems 1, 2 the problems of image P analysis.

3. The problems for space E^N of dimension N can be interpreted as follows.

1) The space E^1 i.e. the line. The objects for it are intervals given on the line and intersection by 2, 3 and so on till n (n – general number of intervals) is possible. We must find the part of the line E_k^1, where sharp k of any intervals intersect, $k=\overline{0,n}$ (problem 1) or find domains of the line $E_k^1(i_1,...,i_k)$ where k concrete intervals $i_1,...,i_k$ intersect, $(k=\overline{1,n})$ (problem 2).

2) Space E^2, i.e. flatness. The objects for it are given on it rectangles, which sides are paralleled to coordinate axes and intersection by 2, 3 an n is possible. The choice of rectangles as interacting objects evidently does not limit the community of considering, because any flat figure we can submit with any degree of exactness with the help of the inscribed in it "tightly packed" rectangles. We should find domain on the flatness E_k^2, where sharp k of any rectangles intersect, $k=\overline{0,n}$ (problem 1) or find fields on the flatness $E_k^2(i_1,...,i_k)$ where k of the concrete rectangles intersect $i_1,...,i_k$, $k=\overline{1,n}$ (problem 2).

3) Space E^3, i.e. three-dimensional space. The objects for it are given in it right parallelepipeds with the sides which are paralleled to coordinate axes. Intersection of the parallelepipeds by 2, 3, an n is possible. The choice of the right parallelepipeds as the objects in the three-dimensional space does not limit the community, because any three-dimensional object can be submit with the help of the "tightly packed" right parallelepipeds inscribed in it. We should find domains E_k^3 in the space E^3, where sharp k parallelepipeds intersect indifferently which ones, $k=\overline{0,n}$ (problem 1) or find domains $E_k^3(i_1,...,i_k)$ in E^3 where k concrete parallelepipeds $i_1,...,i_k$, $k=\overline{1,n}$ intersect (problem 2).

4. Let's consider the general case of the problems 1, 2 of image P analysis in the space E^N of the optional dimension N. According to (1) the optional image P we get from intersection images P_k $(k=\overline{0,n})$ of sharp k of any different objects and domains E_k^N of intersections P_k existing for different k according (3) do not intersect. According to this let's introduce the system of binary functions $f_k(x), k=\overline{0,n}$, in the space E^N.

$$f_k(x)=\begin{cases}1, x\in E_k^N;\\ 0, x\notin E_k^N.\end{cases} \tag{5}$$

As we can see from (5) the optional k-th function $f_k(x)$ takes meaning 1 in the domain E_k^N, where intersection P_k exists, and meaning 0 in others spaces where this intersection does not exist. So it would be naturally to call function $f_k(x)$ the k-th normal spectral function of image $P(1)$, and the collection of all such functions $f(x)=\{f_0(x),...,f_n(x)\}$ – the normal spectrum of point image. Analogically, according to (2) an optional intersection P_k we get from intersections $P_k(i_1,...,i_k)$ of sharp k different concretely determined objects $i_1,...,i_k$ and the domains $E_k^N(i_1,...,i_k)$ of intersection $P_k(i_1,...,i_k)$ existing for different combination of objects $(i_1,...,i_k)$ according to (4) do not intersect. According to it let's introduce the system of binary functions $G(x \mid i_1,...,i_k)$, $k=\overline{1,n}$, $i<i_1<\mathrm{K}\ i_k \le n$ in space E^N, determined in the form.

$$G(x \mid i_1,...,i_k)=\begin{cases}1, x\in E_k^N(i_1,...,i_k);\\0, x\notin E_k^N(i_1,...,i_k).\end{cases} \tag{6}$$

From (6) we see that the optional k-th function $G(x \mid i_1,...,i_k)$ is equal 1 in domain $E_k^N(i_1,...,i_k)$ where intersection $P_k(i_1,...,i_k)$ exist and is equal 0 in the other domains, where this intersection does not exist. In according to this let's call given function the k-th marked spectral function of P image. The collection of all such functions for all possible combinations of objects by $1,2,...,n$ let's call marked spectrum of image P and designate it $G(x)$. So in contrast to normal spectrum some spectral functions of which display the domains of space where concrete image's combinations intersection is depicted. But both have the property of indication of domains with all possible typical fragments of given image. It prompts us the idea for solving Problems 1 and 2, considered in the next point.

5. As all objects in the space E^N are N-dimensional right parallelepipeds in E^N with the sides paralleled to N coordinate axes, the intersection of any number k objects ($k=\overline{1,n}$) is N-dimension right parallelepiped in E^N with the sides paralleled to N coordinate axes. So any non-concretized P_k or concretized $P_k(i_1,...,i_k)$ by structure intersection of k objects could be projected on all coordinate axes $x_1,...,x_N$:

$$P_k \Rightarrow [P_k(x_1),P_k(x_2),...,P_k(x_N)],\ k=\overline{1,n}; \tag{7}$$

$$P_k(i_1,...,i_k)\Rightarrow [P_k(x_1 \mid i_1,...,i_k),P_k(x_2 \mid i_1,...,i_k),...,P_k(x_N \mid i_1,...,i_k)],\ k=\overline{1,n}, \tag{8}$$

where $P_k(x_i)$ is the project of objects P_k intersection on the axe x_i, and $P_k(x_i \mid i_1,...,i_k)$ is the project of objects $P_k(i_1,...,i_k)$ on the same axe. The difference between this projects is the same as in initial intersections: in the project $P_k(x_i)$ is pointed only number k of objects in initial intersection and in project $P_k(x_i \mid i_1,...,i_k)$ is given else concrete list $(i_1,...,i_k)$ of objects in intersection. Any intersection P_k according to (2) consists in general case of C_n^k distributed in the space intersections $P_k(i_1,...,i_k)$ of concrete objects $(i_1,...,i_k),(k=\overline{1,n})$. So the project of the intersection P_k on any axis consists as well in general case from C_n^k distributed along this axis projects on it of intersection $P_k(i_1,...,i_k)$:

$$P_k(x_r)=\bigcup_{i_1<i_2<\mathrm{K}<i_k} P_k(x_r \mid i_1,...,i_k), k=\overline{1,n}, r=\overline{1,N}.$$

This is the second distinction between projects (7) and (8). The likeness between them is: we have one-valued adequacy of objects $P_k(i_1,...,i_k)$ any intersection and its projects and one-valued adequacy of objects P_k any intersection and its projects at the lack in general case one-valued reciprocal correspondence. However in the case $N=1$ (i.e. in one-dimensional space) we have one-to-one correspondence between objects intersections and their projects on the only in this axis. The introduced objects of N-dimensional objects intersections – normal $P_k(x_r)$ and marked $P_k(x \mid i_1,...,i_k)$ are one-dimensional objects with which it is much easier to work. For them the general N-dimensional spectral function – normal (5) and marked (6) jump in according spectral functions from one variable:

$$f_k(x_r)=\begin{cases}1, & x_r\in E_{rk}^1,\\0, & x_r\notin E_{rk}^1,\end{cases}\ k=\overline{0,n},\ r=\overline{1,N}; \tag{9}$$

$$G_k(x_r \mid i_1,...,i_k) = \begin{cases} 1, & x_r \in E^1_{rk}(i_1,...,i_k), \\ 0, & x_r \notin E^1_{rk}(i_1,...,i_k), \end{cases} \quad k = \overline{1,n}, r = \overline{1,N}. \tag{10}$$

According to (9) the k-th function $f_k(x_r)$ takes the value 1 in that part E^1_{rk} of axis x_r, where the project $P_k(x_r)$ of intersection P_k exists and the value 0 in the other parts where the project is absent. So the function $f_k(x_r)$ could be called the k-th one-dimensional normal spectral function of the intersection P_k project $P_k(x_r)$ on the axis x_r. Analogically, according to (10), the k-th function $G_k(x_r \mid i_1,...,i_k)$ is equal 1 in that part of $E^1_{rk}(i_1,...,i_k)$ of the axis x_r, where the project $P_k(x_r \mid i_1,...,i_k)$ of intersection $P_k(i_1,...,i_k)$ exists and is equal 0 in the other parts where this project is absent. So the function $G_k(x_r \mid i_1,...,i_k)$ can be called the k-th marked one-dimensional spectral function of the intersection $P_k(i_1,...,i_k)$ projects $P_k(x_r \mid i_1,...,i_k)$ on the axis x_r.

Let's call the collection of all functions $f_k(x_r), k = \overline{0,n}$ the normal one-dimensional spectrum of image $P(1)$ along axis x_r of N-dimensional space $(r = \overline{1,N})$. Analogically the collection of all functions $G_k(x_r \mid i_1,...,i_k), k = \overline{1,n}$ can be called the marked one-dimensional spectrum of the image $P(1)$ along axis x_r of N-dimensional space $(r = \overline{1,N})$.

Introduced one-dimensional spectra have the property of indication of the corresponding axes N-dimensional space domains, where the project of considered intersection of the objects in the space is. From this follows that Problems 1 and 2 of the image P in the N-dimensional space analysis rationally to try to solve by the reducing them to the one-dimensional spectrum projects of this image on all N axes of the space analysis or simply to calculating one-dimensional spectra along paralleled lines intersecting the image.

6. The realization of idea given in p. 5. Let's begin from the analysis of the images in the one-dimensional space i.e. on the line. The problem 1 in this case includes the following. The image P in the form of the n one-dimensional objects – closed intervals $[a_i, b_i], i = \overline{1,n}$ on the axis x is given (fig.1). This intervals can interact forming different intersection. As a result the image P disintegrates accordingly to (1) into the images: P_0 in the domain E^1_0 on the axis x, which does not contain any intervals; P_1 in the space E^1_1, containing only any single non-intersecting with the others intervals; P_2 in the domain E^1_2, containing only pairwise intersecting intervals, ... P_n in the domain E^1_n containing only intersection of all intervals. The problem consists in finding domains $E^1_k, k = \overline{0,n}$ on the axis x, containing the images $P_k, k = \overline{0,n}$ in the form any fixed number k intersection of non-concretized (i.e. any) intervals.

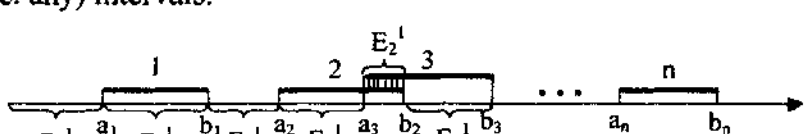

Fig. 1

Let interpret the axis x points as the moments of time. Then the problem 1 with the given system of intervals can be put to accordance to the mathematical model in the form of the finite automata without memory (fig. 2) with n binary inputs $x_1,...,x_n$, $x_i \in \{0,1\}$ on which single impulses $1(a_i, b_i)$ are being given (by one on every input), existing in the time intervals (a_i, b_i) according to given intervals $(i = \overline{1,n})$ .This automaton has one binary output $y, y \in \{0,1\}$, on which the fundamental symmetrical index $p(p = \overline{0,n})$ Boolean function F_n^p of n input variables $x_1,...,x_n$ is being realized. So, the automaton output variable is being expressed with the help of its input variables in the form

$$y^p = F_n^p(x_1,...,x_n);\ y, x_i \in \{0,1\}. \tag{11}$$

As it is known [4] the fundamental symmetrical Boolean function $F_n^p = 1$ only when p of any its arguments are equal 1 and $F_n^p = 0$ in the rest of cases.

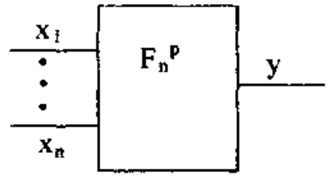

Fig. 2

So, the output variable of automaton model $y = 1$ only in that case when p of any its input variables x_i, are equal 1. It means that on the output of this automaton the single impulses are being yielded in those time intervals in which p input single impulses act simultaneously. So (look at (5), (9)) automaton-model (look at fig. 2) with the realized Boolean function F_n^p yields in the out the normal one-dimensional spectral function $f_p(x)$ of one variable x, marking with its single values those domains of the axis x where p of n given intervals $[a_i, b_i], i = \overline{1,n}$ are being intersected. Varying the value of index p of the automaton-model function F_n^p from 0 to n we'll get the normal one-dimensional spectrum $f(x) = \{f_0(x), f_1(x),..., f_n(x)\}$ components of which are functions marking the domains of the axis x where any possible number p $(p = \overline{0,n})$ of the given intervals $[a_i, b_i]$ intersect. This is the solution of the Problem 1 of analysis of the image in the one-dimensional space.

The solution of the Problem 2 of the image in the one-dimensional space analysis differs from given by the choice of the automaton-model function in form

$$y = \Phi_n^k(x_1,...,x_n) = (x_{i_1},...,x_{i_k})(\overline{x_{i_{k+1}} \vee K \vee x_{i_n}}), x_i, y \in \{0,1\}. \tag{12}$$

The solution of problems 1, 2 of the images in the two- and three-dimensional space analysis with the help of section method is being reduced to the solution of this problems in one-dimensional space.

References

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