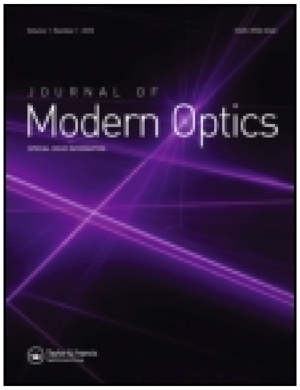

Journal of Modern Optics
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tmop20

Relaxation-induced stabilisation of optical pulsed vortex beams in media with a composite cubic-quintic nonlinearity

R.A. Vlasov [a] , V.M. Volkov [b] , A.A. Stankevich [b] & E.V. Doktorov [a]

[a] Stepanov Institute of Physics, National Academy of Sciences of Belarus, B.I., 68 Nezavisimosti Av., Minsk 220072, Belarus

[b] Belarusian State University, 4 Nezavisimosti Av., Minsk 220050, Belarus

Available online: 11 Apr 2011

Journal of Modern Optics
Vol. 58, No. 9, 20 May 2011, 766–771

Taylor & Francis
Taylor & Francis Group

Relaxation-induced stabilisation of optical pulsed vortex beams in media with a composite cubic–quintic nonlinearity

R.A. Vlasov[a], V.M. Volkov[b]*, A.A. Stankevich[b] and E.V. Doktorov[a]

[a]*Stepanov Institute of Physics, National Academy of Sciences of Belarus, B.I., 68 Nezavisimosti Av., Minsk 220072, Belarus;* [b]*Belarusian State University, 4 Nezavisimosti Av., Minsk 220050, Belarus*

(*Received 31 December 2010; final version received 3 March 2011*)

The problem of the generation of stable (quasi-stable) 3+1D light bullets in nonlinear media is addressed. We consider a medium with a two-component relaxing cubic nonlinearity, and non-relaxing quintic one. It is shown that a special adjustment of the pulse duration and the parameters of the two-component relaxing nonlinearity enables one not only to suppress distortions of the temporal pulse envelope and self-induced frequency shift, but to suppress a destructive effect of the azimuthal instability of vortex pulsed beams as well.

Keywords: singular light beams; relaxing nonlinearity; Raman effect

1. Introduction

The formation of stable spatiotemporal localized structures of light fields (light bullets), when intensive laser radiation propagates in nonlinear media, is one of the most urgent problems, which concerns the current theory of solitons and possible applications [1–3]. The latest results on the generation of light bullets can be found in [4,5]. In this context, of special interest are vortex-like pulsed beams (singular optical bullets) which have attracted the attention of researchers because of some specific features, such as the stable tubular form of the vortex beams. However, it is convincingly shown theoretically and experimentally that optical vortices, in the most practically usable cases, are inherently unstable against azimuthal perturbations [6,7].

One of the basic conditions for the formation of stable optical vortex bullets (spatiotemporal solitons) is the presence of the focusing nonlinearity saturation preventing the spatiotemporal collapse of radiation [1–3]. The most extensively studied models include a combination of the cubic and quintic nonlinearities with the opposite signs as well as a saturable nonlinearity expressed in the rational form. The stabilizing effect of the polynomial nonlinearity, which results from the competition between the cubic and quintic components, not only prevents the collapse, but mostly suppresses the spatiotemporal instability of pulsed beams as well, when the localized energy is significantly higher than the critical value required for the compensation for the dispersion and diffraction. In this case, the analytical estimates and numerical simulations are shown the possibility of stable propagation of optical pulsed vortices with the intensity close to the saturation threshold [8]. It should also be noticed that in the case of a nonlocal nonlinearity [6,9] and bimodal systems with hidden vorticity [10], the stability of optical vortices is improved.

During the propagation of ultrashort pulses in nonlinear media, when the nonlinearity relaxation time is comparable with the pulse duration, the non-stationary response of the media exerts a destructive effect on the soliton-like pulse stability. However, it is remarkable that in media with two-component relaxing cubic nonlinearity where the relaxation is accompanied by a frequency shift of Raman type, the influence of the fast response of the focusing component can be compensated for by the slow response of the defocusing component. The condition of the mutual compensation (counterbalance) of perturbations of the relaxation origin has been obtained for the first time in [11] for the case of temporal solitons. It was demonstrated that for a soliton pulse whose duration is longer than the fast relaxation time and shorter than the slow relaxation time, the soliton envelope distortion related to the fast non-stationary response is essentially compensated by a relatively slow nonlinear response. Recently the concept of fast and slow interplaying nonlinearities was extended to the $3+1$D case, and a possibility to generate light bullets in reorientational nematic liquid crystals was proven [12].

*Corresponding author. Email: volkovvm@bsu.by

ISSN 0950–0340 print/ISSN 1362–3044 online

DOI: 10.1080/09500340.2011.569858
http://www.informaworld.com

The aim of the present paper is to show that the approach with two relaxing nonlinearities can be developed for spatiotemporal solitons as well while considering *singular* pulsed beams. Namely, the competition between the fast focusing and slow defocusing nonlinearities is capable of suppressing the self-induced frequency shift and azimuthal instability of optical pulsed vortices.

2. Theoretical model

The $3+1$D nonlinear extended Schrödinger equation governing the pulse propagation in a media with the cubic–quintic nonlinearity including two relaxing cubic components in its general form is

$$\mathrm{i}\frac{\partial E}{\partial z}+\Delta_\perp E+\frac{\partial^2 E}{\partial t^2}+|E|^2E-|E|^4E - \left(\alpha\frac{\partial|E|^2}{\partial t}+\beta\int_{-\infty}^{t}|E|^2\exp[(t'-t)\tau_s^{-1}]\,\mathrm{d}t'\right)E=0, \quad (1)$$

$$-T_0<t<T_0,\quad r=(x^2+y^2)^{1/2}<R$$

where the dimensionless quantities are introduced, which were used before in [1,8]. Here, as usual, $E=E(r,\phi,t,z)$ is the complex envelope of the electric field normalized to the threshold of the nonlinearity saturation, τ_s is the relaxation time of the slow component normalized to the character pulse duration $\tau_0=10^{-12}$ s, $\Delta_\perp E=r^{-1}\partial_r(r\partial_r E)+r^{-2}\partial_\phi^2 E$, and the parameters α and β characterize the non-stationary response of the fast focusing and slow defocusing components, respectively [11]. It is also supposed that the relaxation time τ_s of the slow component exceeds the pulse duration whereas the relaxation time τ_f of the fast component is essentially less than the pulse width τ: $\tau_s>\tau\gg\tau_f$. In particular, in the case of picosecond pulses, relaxation times of the fast and slow components are supposed to be in a range of 10^{-14}–10^{-13} and 10^{-12}–10^{-11} s, respectively.

In compliance with the special features of optical vortices, the initial conditions for Equation (1) are specified as:

$$E(r,\phi,t,0)=A(r,t)\exp(\mathrm{i}m\phi). \quad (2)$$

The real function $A(r,t)$ stands for the spatiotemporal profile of the field amplitude, which is of a specific toroidal form:

$$A(r,t)=A_0(\rho)\sin(\vartheta),\quad \rho=(r^2+t^2)^{1/2},\quad \sin\vartheta=r/\rho, \quad (3)$$

where $A_0=A_0(\rho)$ is a solution of the two-point boundary value problem

$$\frac{1}{\rho^2}\frac{\partial}{\partial\rho}\rho^2\frac{\partial A_0}{\partial\rho}-\frac{(m^2+1)}{\rho^2}A_0-\kappa A_0+(A_0^2-A_0^4)A_0=0,\quad \lim_{\rho\to0}A_0(\rho)=\lim_{\rho\to\infty}A_0(\rho)=0. \quad (4)$$

Here κ is the propagation constant.

The complex exponential multiplier in (2) accounts for the linear circular phase modulation, and the integer constant m is the topological charge of a vortex beam, associated with the azimuthal field moment.

Initial-boundary conditions (2)–(4) are chosen to fit the axial symmetry of a pulsed beam with the nested vorticity in the transverse plane. In this case the pulse width depends on κ and $\tau\cong10$ at $m=\pm1$, $\kappa=0.143$. In the numerical simulations, the boundary conditions $E(-T_0)=E(T_0)$, $E(R_0)=0$ were used and a dissipation was added at the boundary area of the calculation domain to prevent the boundary effects.

In this formulation, the problem (1)–(2) has been studied in close detail in [8]. The authors of [8] have obtained families of approximate quasi-stationary solutions with the non-zeroth topological charge and have conducted a linear analysis of their stability against azimuthal perturbations. The results obtained in [8] adequately predict the dynamics of the azimuthal perturbations, and what is the most interesting is that for $m=\pm1$ a stability region in the input parameters' area is determined. The results of such an analysis are qualitatively confirmed by directly solving the problem over a sufficiently extended distance of propagation.

3. Condition for self-compensation of the relaxing perturbation

It is known (see, for example [1]) that the influence of the inertiality manifests itself in the self-induced frequency shift (the Raman effect). The direction of the frequency shift is determined by the nonlinearity sign. This shift is toward the low-frequency domain for the focusing nonlinearity and vice versa for the defocusing one. The competition of the relaxing nonlinearities with opposite signs is just what enables the mutual compensation to be realized to prevent the pulse spectrum from a change on the whole. For the case of temporal soliton propagation, the conditions of practically complete suppression of nonlinear perturbation are obtained in [11]. These conditions include the relationship between the pulse duration, relaxation times and the nonlinear refractive indices. This allows us to argue that there is the feasibility of suppressing the self-induced frequency shift by using two opposite nonlinearities: the fast and slow ones, while controlling the pulse spectrum. This idea seems to be realizable in the case of the presence of azimuthal perturbations of optical vortex pulses because we are dealing here again with the problem of controlling the pulse spectrum.

To determine the conditions of the mutual compensation for nonlinear distortions, we fix the values of α and τ_s, whereas the value of β should be found from the condition of minimizing the absolute value of the functional $\psi(E) = \iiint tr|E|^2 \, dt \, d\phi \, dr$. This functional was calculated using the result of the numerical solution of Equation (1) with initial conditions for which $\psi(E(z=0)) = 0$. The condition $\psi(E) = 0$ is fulfilled for symmetric pulses with the maximum at the point $t = 0$ and $E(t) = E(-t)$. The value and sign of the functional $\psi(E)$ characterise the pulse shift in time, which is due to the frequency shift in the presence of the group velocity dispersion. It is evident that the absence of the frequency shift ensures the preservation of the zero value of the functional $\psi(E)$ and the condition of the minimum of the absolute value of $\psi(E)$ at a sufficiently long distance of the pulse propagation is equivalent to the condition of the minimum of the overall frequency shift stemming from the influence of inertiality of the two-component nonlinearity. Thus, the problem of determining the parameter β related to the feasibility of the mutual compensation of the frequency shift is reduced to the classical one-parametric optimization problem. The use, for instance, of standard optimization methods provided by the Matlab optimization toolbox permitted us to find a suitable value of β using as an objective function the above functional calculated for the solution of Equation (1) at a comparatively short distance $z = 50$–100.

The result of the numerical simulations for $\kappa = 0.143$, $m = \pm 1$, $\tau_s = 50$ are shown in Figure 1. In this case, in the absence of perturbations ($\alpha = 0$, $\beta = 0$), there occurs the formation of a stable vortex structure of a pulsed beam having unit topological charge. The dependence of the absolute value of the functional $\psi(E)$ on the parameter β at $\alpha = 0.1328$ is presented in Figure 1(*a*). As is seen from the results presented, the minimum value of this functional is achieved at $\beta \cong 0.2878$.

Figure 1(*b*) illustrates also the spectra of the vortex pulsed beam at $z = 1000$ in the presence of one of the perturbing components (fast or slow one) and in the presence of both components simultaneously at the condition of the minimum of $|\psi(E)|$. It is easily seen that under the action of each of the perturbation components individually there takes place a shift of the pulse frequency, the absolute value of which exceeds the width of its initial spectrum. The fast (focusing) component brings about the frequency shift toward the low frequency region, whereas the slow (defocusing) one is responsible for a shift being much the same in value but toward the high-frequency region. In combination, the influence of both components results in mutual suppression of the frequency shift, and the pulse spectrum is practically unchangeable on the frequency axis in the process of propagation over a long distance.

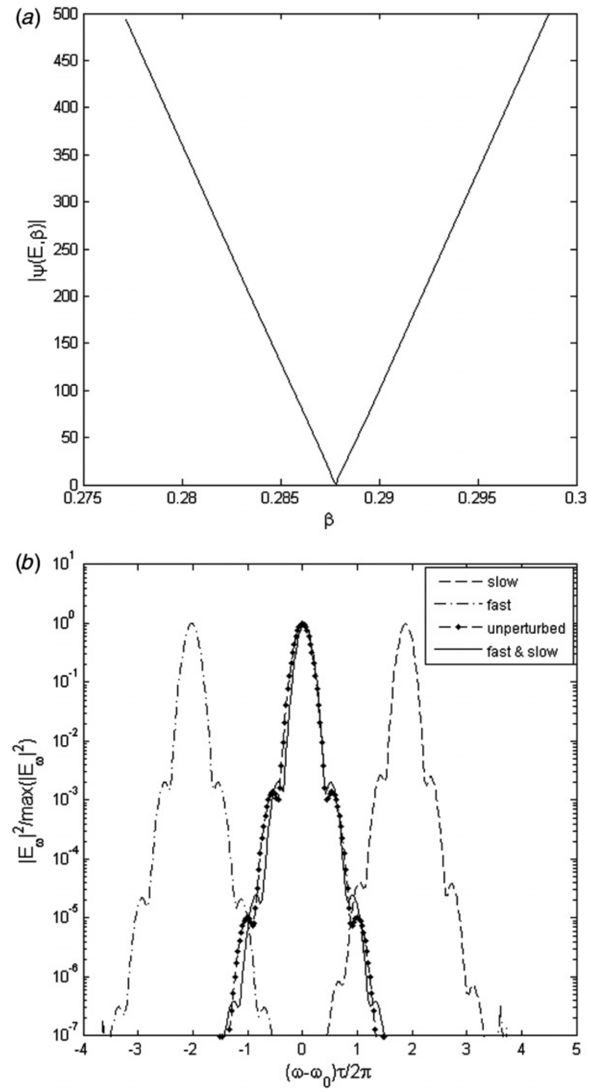

Figure 1. Absolute value of the functional $\psi(E)$ as a function of the parameter β at $\alpha = 0.1328$ and the spectra of the vortex pulsed beam at propagation distance $z = 1000$ in the presence and absence of the relaxing components: slow ($\beta = 0.2878$, $\alpha = 0$), fast ($\alpha = 0.1328$, $\beta = 0$), fast and slow ($\alpha = 0.1328$, $\beta = 0.2878$), unperturbed ($\beta = 0$, $\alpha = 0$).

4. Influence of relaxation perturbation on stability of optical vortex pulsed beams

As criteria for estimating the influence of relaxation perturbation on stability of vortex pulsed beams, we used the energy integral

$$P(z) = \sum_k \int_r r \int_t \int_\phi \left| E_k(r,\phi,z,t) \right|^2 \mathrm{d}\phi \, \mathrm{d}t \, \mathrm{d}r \qquad (5)$$

and the relative value of azimuthal perturbations

$$P_s(z) = P^{-1}(z) \times \sum_{s \neq m} \int_r r \int_t \left| \int_\phi E_k(r,\phi,z,t) \exp(\pm \mathrm{i} s\phi) \, \mathrm{d}\phi \right|^2 \mathrm{d}t \, \mathrm{d}r. \qquad (6)$$

A persistent rise of azimuthal perturbations as well as a decrease in the energy integral, are associated with the instability, which results in the destruction of the vortex structure of the pulsed beam at a sufficiently long distance of propagation.

Let us consider, at first, the case of the initial conditions (2–4) with $\kappa = 0.143$, $m = 1$, which correspond to the stable propagation of the singular pulse in the absence of the inertiality of the nonlinear response ($\alpha = 0$, $\beta = 0$) [8]. The initial stage of evolution of such a beam at $z < 200$ is accompanied by the transient process, and, as a result of this, there occurs the scattering of some initial energy (about 2–3%) and formation of a localized quasi-stationary vortex structure characterized by a relative stability of the energy integral and the absence of rise in the azimuthal perturbation. When one of the perturbing components (fast or slow) is switched on, the situation is changed toward loss of the stability, which manifests itself by a stronger scattering of the pulse energy (Figure 2(*a*)) and the rise in the azimuthal perturbations (Figure 2(*b*)). It is remarkable that under the simultaneous effect of both components, when magnitudes of the perturbing factors α and β satisfy the Raman shift compensation, the dynamics of the singular pulsed beam remains stable as in the absence of the relaxation perturbations (see Figure 2).

Of obvious interest is also the investigation of the influence of the perturbing factors on the dynamics of vortex pulsed beams in the case where they are not stable in the absence of the relaxation induced perturbations. As an example, let us consider the solution of problem (2)–(4) at $\kappa = 0.15$, $m = 2$. As is known [8], vortex pulsed beams with the topological charge $|m| > 1$ are azimuthally unstable. The development of the instability is excited by the exponential rise of azimuthal perturbations (6). It is remarkable that in the presence of the relaxation factors, satisfying the conditions of the complete or partial compensation for the frequency shift, the rise in azimuthal perturbations is essentially retarded. As a result the distance of stable propagation of a vortex-structured beam (before the filamentation) increases by 20–40% as compared to the case of the non-perturbed propagation in media with the instantaneous nonlinear response (Figure 3).

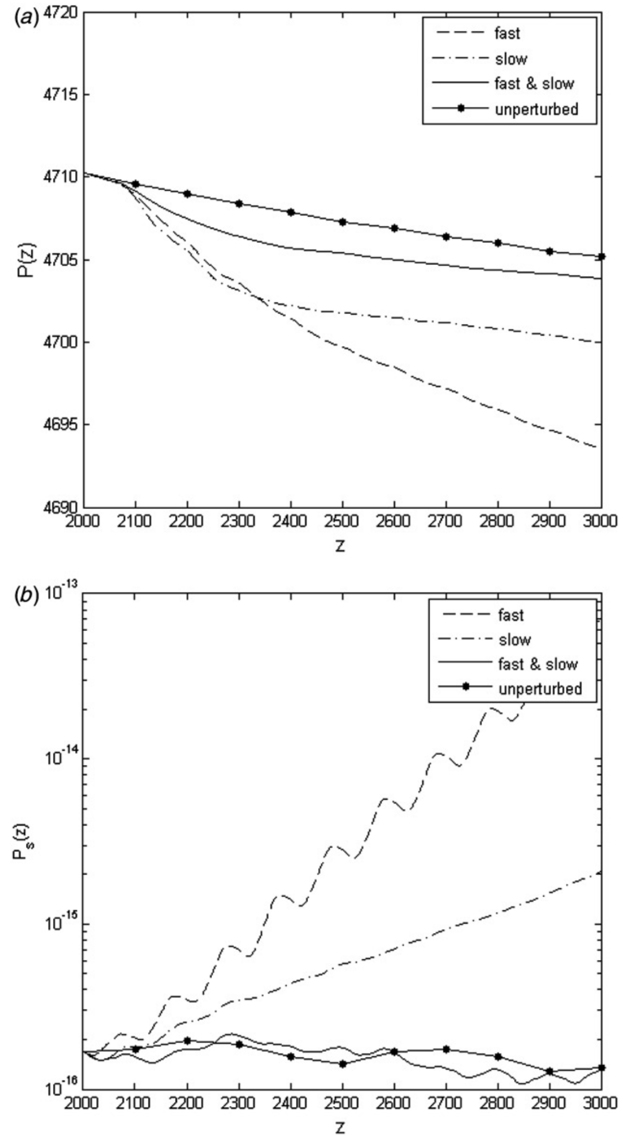

Figure 2. Dynamics of the energy integral $P(z)$ and azimuthal perturbation $P_s(z)$ in the presence and absence of the relaxing components: slow ($\beta = 0.2878$, $\alpha = 0$), fast ($\alpha = 0.1328$, $\beta = 0$), fast and slow ($\alpha = 0.1328$, $\beta = 0.2878$), unperturbed ($\beta = 0$, $\alpha = 0$).

5. Conclusions

We have established an effective suppression of azimuthal perturbations during the propagation of

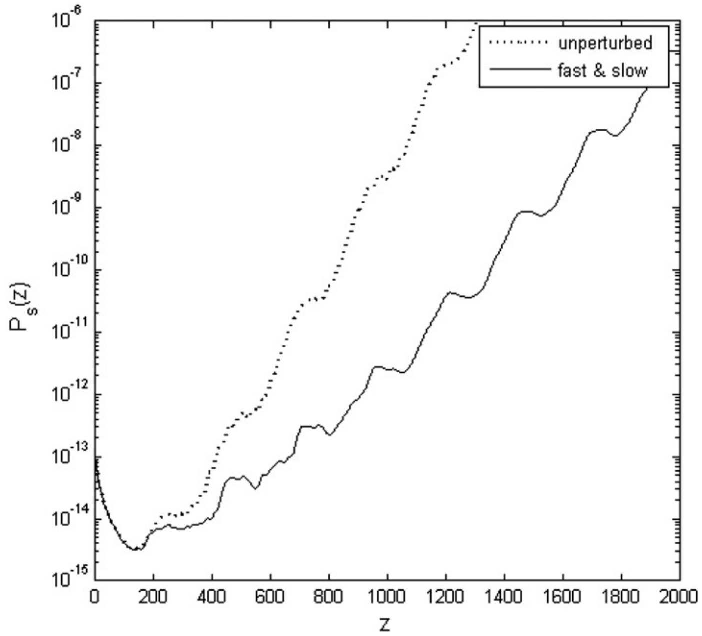

Figure 3. Dynamics of the azimuthal perturbation of the vortex pulsed beam with initial conditions (2)–(4) at $m = 2$, $\kappa = 0.15$ in the presence and absence of the relaxing components: unperturbed ($\beta = 0$, $\alpha = 0$), fast and slow components are under conditions of partly suppressed self-induced frequency shift ($\alpha = 0.2$, $\beta = 0.3662$).

pulsed vortex beams in media with a composite cubic–quintic nonlinearity, in which there are two cubic components with opposite signs and different relaxation times (slow defocusing and fast focusing components of the cubic nonlinearity). A non-trivial nature of this phenomenon lies in the fact that each of the two components exerts a destructive effect on the stability when acting individually, whereas it is just the joint action of these components that improves stability of optical vortices, allowing the azimuthal perturbations to be suppressed completely or partly. The full suppression of the azimuthal perturbations is observed in the case of the optical vortices demonstrating stable behavior in the medium with instantaneous nonlinear response. In the case of the topological charge $|m| \geqslant 2$, when stable vortex states can not be implemented in the media with the instantaneous cubic–quintic nonlinearity, a partial suppression of the azimuthal instability is achieved. This enables one to prolong the propagation distance by 20–40% as compared to the propagation of the same pulsed vortex in media with the instantaneous nonlinear response. The condition of maximum suppression of the azimuthal perturbations is well consistent with the condition of suppression of the self-induced frequency shift. Namely, it is necessary that the pulse duration be intermediate between the relaxation times.

At present there are a number of nonlinear optical materials whose properties are described well in the framework of the cubic–quintic model. To mention a few, we can refer to some semiconductors and doped glasses (for example, AlGaAs and CdS_xS_{1-x}), as well as to chalcogenide glasses and some organic materials (e.g. stilbazolium derivatives).

Acknowledgements

This work was partly supported by Belarussian Republican Foundation for Fundamental Research (Grant F10R-103).

References

[1] Kivshar, Yu.S.; Agrawal, G. *Optical Solitons: From Fibers to Photonic Crystals*; Academic: San Diego, 2003.
[2] Akhmediev, N.N.; Ankiewicz, A. *Solitons, Nonlinear Pulses and Beams*; Chapman & Hall: London, 1997.
[3] Malomed, B.A.; Mihalache, D.; Wise, F.; Torner, L. *J. Opt. B: Quantum Semiclassical Opt*. **2005,** *7,* R53–R72.
[4] Torner, L.; Kartashov, Ya.V. *Opt. Lett*. **2009,** *34,* 1129–1131.
[5] Lobanov, V.E.; Kartashov, Ya.V.; Torner, L. *Phys. Rev. Lett*. **2010,** *105,* 033901.
[6] Kruglov, V.I.; Logvin, Yu.A.; Volkov, V.M. *J. Mod. Opt*. **1992,** *39,* 2277–2291.

[7] Tikhonenko, V.; Christou, J.; Luther-Daves, B. *J. Opt. Soc. Am.* **1995,** *12,* 2046–2052.
[8] Mihalache, D.; Mazilu, D.; Crasovan, L.-C.; Towers, I.; Buryak, A.V.; Malomed, B.A.; Torner, L.; Torres, J.P.; Lederer, F. *Phys. Rev. Lett.* **2002,** *88,* 073902.
[9] Mihalache, D.; Mazilu, D.; Lederer, F.; Malomed, B.A.; Kartashov, Ya.V.; Crasovan, L.-C.; Torner, L. *Phys. Rev. E* **2006,** *73,* 025601.
[10] Desyatnikov, A.S.; Mihalache, D.; Mazilu, D.; Malomed, B.A.; Denz, C.; Lederer, F. *Phys. Rev. E* **2005,** *71,* 026615.
[11] Afanas'ev, A.A.; Doktorov, E.V.; Vlasov, R.A.; Volkov, V.M. *Opt. Commun.* **1998,** *153,* 83–89.
[12] Burgess, I.B.; Peccianti, M.; Assanto, G.; Morandotti, R. *Phys. Rev. Lett.* **2009,** *102,* 203903.