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Penetration of a pyramid indenter into a multilayer coating

A simple model of elastic–plastic penetration of a Berkovich pyramid into a multilayer coating has been developed. The model of a simply deformable body was used. It generalizes Winkler's foundation to the case of elastic–plastic deformation of each layer. The elastic–plastic deformation of a layer was modeled by Prandtl's bilinear approximation of a stress–strain curve. The homogenized coefficients of the elastic and plastic properties of the layer set and an analytical solution of the problem were found. The distribution of contact stress and depth of a pyramid indenter penetration were obtained for different angles.

Keywords: Elastic–plastic deformation; Bilinear approximation of a stress–strain curve; Contact stress; Berkovich indenter

1. Introduction

Sometimes, the plastic deformation of machine parts should be taken into account in their design. These problems are usually solved by methods of the theory of plasticity. However, an analytical solution cannot be obtained in most cases. Therefore, simplified models of the deformation of a rigid body are elaborated in addition to the mathematical theory of plasticity devoted to the methods of an exact solution of the problem. The simplified models are based on specific hypotheses and assumptions [1].

In particular, difficulties in the definition of the contact stresses in the mechanics of rigid bodies are defined by the fact that the displacement in a fixed point of the contact surface depends on the contact pressure distribution in the whole contact area. Hence the definition of an analytical solution of the contact problem requires a solution of the integral equation or truncation of a system of linear equations for the definition of a series of coefficients [2].

The difficulties are removed when the modeled body (coating on a rigid surface) consists of rod elements deformable only in the direction of penetration of the indenter. The size of the rod cross-section is neglected in comparison with the area of contact. This model is named as Winkler's foundation in the case of elastic rod deformation [2].

A simple model of elastic–plastic penetration of the Berkovich pyramid into a multi-layer coating was developed in the paper. A model of a simply deformable body was used. It generalizes Winkler's foundation to the case of elastic-plastic deformation of each layer. The elastic–plastic deformation of a layer was modeled by Prandtl's bilinear approximation of a stress–strain curve [3]. The homogenized coefficients of the elastic and plastic properties of the layer set were found. An analytical solution of the problem was found. The distribution of contact stress and depth of a penetrating pyramid indenter was obtained for different angles of the indenter tip.

2. Generalization of Winkler's foundation to the case of elastic–plastic deformation of a multilayered coating

It is supposed that the surface of the composite coating is plane. This means that the surface deviations are neglected in comparison with the depth of the pyramid penetration. The multilayered coating covers a smooth rigid half space. Furthermore, it is supposed for the creation of a deformation model of a multilayer coating that it can be replaced by rods with a constant square cross-section of width Δ and height h (Fig. 1). The width Δ is neglected in comparison with the radius a of the entered circle for the contact triangle (Fig. 2). The rods can be deformed only in the z-direction and the rod stress state is uniform.

A rod in a simplest multilayered model consists of n subrods (layers) with heights $h_{k,k=\overline{1,n}}$ (Fig. 1). It is supposed that the deformation curve of each layer material in tension (or compression) is defined by Prandtl's bilinear approximation [3]. It means that the mechanic of properties E_k (elasticity modulus), $E_{\text{yield},k}$ (tangent modulus for plasticity in tension), and $\sigma_{y,k}$ (yield stress in tension) are known for fixed k $(k=\overline{1,n})$. The following equations are satisfied (Fig. 2):

$$E^{\text{incompression}}_{\text{yield},k} = E_{\text{yield},k}, \sigma^{\text{incompression}}_{y,k} = -\sigma_{y,k}\left(k=\overline{1,n}\right) \quad (1)$$

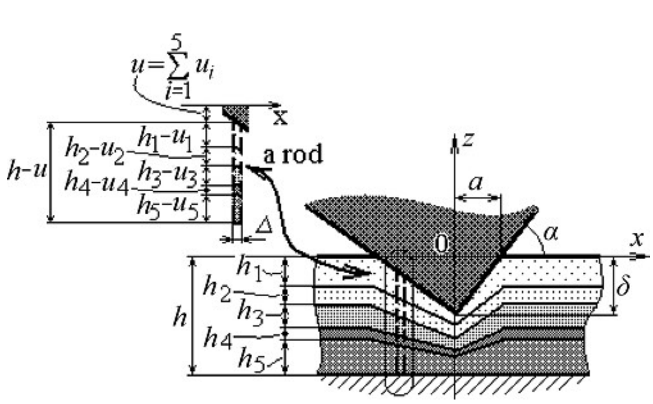

Fig. 1. Deformation of a multilayered coating by a pyramid indenter.

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where $E_{\text{yield},k}^{\text{incompression}}$ $(k = \overline{1,n})$ is a tangent modulus for plasticity in compression and $\sigma_{\text{yield},k}^{\text{incompression}}$ $(k = \overline{1,n})$ is a yield stress in compression.

The stress $\sigma_z(x,y)$ acting in the contact area S on the whole rod with the coordinates $(x,y) \in S$ (Fig. 1) is equal to a corresponding stress for all its subrods (layers). Hence we obtain the following equation for deformation $\varepsilon_{z,k}(x,y)$ in compression (Fig. 2) of a subrod (Fig. 1) with number k $(k = \overline{1,n})$ and coordinates $(x,y) \in S$ (Fig. 3):

$$\varepsilon_{z,k}(x,y) = \begin{cases} \dfrac{\sigma_z(x,y)}{E_k}, \; 0 \geq \sigma_z(x,y) > -\sigma_{y,k} \\ \dfrac{\sigma_z(x,y)}{E_{\text{yield},k}} + \left(\dfrac{E_{\text{yield},k} - E_k}{E_{\text{yield},k} \cdot E_k}\right)\sigma_{y,k}, \; -\sigma_{y,k} \geq \sigma_z(x,y) \end{cases} \tag{2}$$

Taking into account Eq. (2) the displacement in the z-direction $u_{z,k}(x,y)$ for subrod with number k can be defined by the equation (Fig. 1):

$$u_{z,k}(x,y) = h_k \cdot \varepsilon_{z,k}(x,y)$$
$$= \begin{cases} h_k \cdot \dfrac{\sigma_z(x,y)}{E_k}, \; 0 \geq \sigma_z(x,y) > -\sigma_{y,k} \\ \dfrac{\sigma_z(x,y)}{E_{\text{yield},k}} + h_k \cdot \left(\dfrac{E_{\text{yield},k} - E_k}{E_{\text{yield},k} \cdot E_k}\right)\sigma_{y,k}, \; -\sigma_{y,k} \geq \sigma_z(x,y) \end{cases} \tag{3}$$

We define the displacement $u_z(x,y)$ in the z-direction for the whole rod with the coordinates $(x,y) \in S$ by adding up Eq. (3) for $k = \overline{1,n}$ (Figs. 1 and 2):

$$u_z(x,y) = \sum_{k=1}^{n} u_{z,k}(x,y)$$

$$= \begin{cases} h \cdot \dfrac{\sigma_z(x,y)}{E}, \; 0 \geq \sigma_z(x,y) > -\sigma_y \\ h \cdot \dfrac{\sigma_z(x,y)}{E_y} + h \cdot \Omega \cdot \sigma_y, \; -\sigma_y \geq \sigma_z(x,y) \end{cases} \tag{4}$$

Here, the homogenized coefficients of the multilayer coating E, E_y, and σ_y and the weight coefficient Ω are defined by the following equations:

$$E = \left(\sum_{k=1}^{n} \frac{\gamma_k}{E_k}\right)^{-1}, \quad \sigma_y = \sum_{k=1}^{n} \gamma_k \cdot \sigma_{y,k}$$

$$E_y = \left(\sum_{k=1}^{n} \frac{\gamma_k}{E_{\text{yield},k}}\right)^{-1}, \quad \Omega = \sum_{k=1}^{n} \gamma_k \cdot \left(\frac{E_{\text{yield},k} - E_k}{E_{\text{yield},k} \cdot E_k}\right) \tag{5}$$

$$\gamma_k = \frac{h_k}{h}, \quad h = \sum_{k=1}^{n} h_k$$

It is necessary to invert Eq. (4) and obtain the equation for $\sigma_z(x,y)$ of $u_z(x,y)$ for a solution of the contact problem. Thus from Eq. (4) we obtain:

$$\sigma_z(x,y) = \begin{cases} E \cdot \dfrac{u_z(x,y)}{h}, \; 0 \geq \dfrac{u_z(x,y)}{h} > \varepsilon_y \\ E_y \cdot \dfrac{u_z(x,y)}{h} + \Omega \cdot E_y \cdot \sigma_y, \; \varepsilon_y \geq \dfrac{u_z(x,y)}{h} \end{cases} \tag{6}$$

where $\varepsilon_y = (-\sigma_y)/E$.

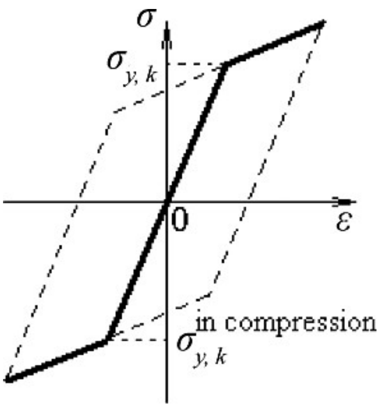

Fig. 2. Deformation curve in tension or compression for the layer material with number k.

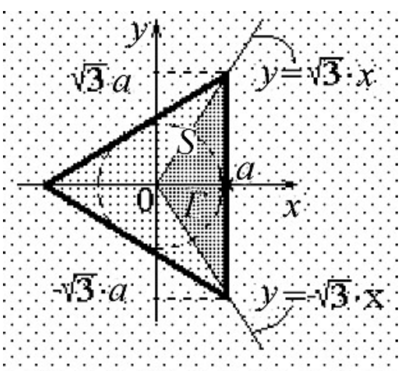

Fig. 3. Contact area of a pyramid indenter.

3. Definition of displacement $u_z(x,y)$ in the contact area

We have to define the size of the contact area with help of a radius a (Fig. 3) and shape of the pyramid surface $f(x,y)$ in the area $\Gamma = \{(x,y)\colon 0 \leq x \leq a, \; -\sqrt{3} \cdot x \leq y \leq \sqrt{3} \cdot x\}$ $(\Gamma \subset S)$ (Fig. 3). It is necessary to emphasize that the surface $f(x,y)$ of a perfect pyramid (touching surface of the coating) is defined in the area Γ by the following equation (Figs. 1 and 3):

$$f(x,y)|_{\Gamma} = \text{tg}(\alpha) \cdot x \tag{7}$$

where α is the angle between a pyramid side and the surface of the coating (Fig. 1).

Therefore, the maximum depth of penetration δ is defined by the equation:

$$\delta = -f(a,0)|_{\Gamma} = -\text{tg}(\alpha) \cdot a \tag{8}$$

Hence

$$u_z(x,y)|_{\Gamma} = f(x,y)|_{\Gamma} + \delta = \text{tg}(\alpha) \cdot (x - a) \tag{9}$$

Taking into account Eqs. (6) and (9) we obtain:

$$\sigma_z(x,y)|_\Gamma = \begin{cases} E \cdot \dfrac{\mathrm{tg}(\alpha)\cdot(x-a)}{h}, & 0 \le \dfrac{\mathrm{tg}(\alpha)\cdot(x-a)}{h} < \varepsilon_y \\ E_y \cdot \dfrac{\mathrm{tg}(\alpha)\cdot(x-a)}{h} + \Omega \cdot E_y \cdot \sigma_y, & \varepsilon_y \le \dfrac{\mathrm{tg}(\alpha)\cdot(x-a)}{h} \end{cases} \tag{10}$$

The spatial distribution of the contact stress looks like a truncated pyramid even for very small indenter penetrations (Fig. 3). This explains the yield flow of material at the tip of the pyramid. The area of "truncation" grows when the depth of indentation increases. The area of "truncation" is a real area of the hardness indentation after unloading. The plastic deformation of the contact area has an overwhelming effect for indenter penetrations more than $0.03 \cdot h$. The contact stress $\sigma_z(x,y)$ has small deviations in comparison with the value of maximum stress; therefore, it looks like a constant.

4. Functional dependence between force and depth of indentation

Since the indenter is a Berkovich pyramid with equal sides (Figs. 1 and 3) the force (P) can be defined by the radius a in S and the contact stress acting on one side, Eq. (10) and Fig. 3:

$$P = -\int_S \sigma_z(s)\,\mathrm{d}s = -3\cdot\int_0^a \left(\int_{-\sqrt{3}\cdot x}^{\sqrt{3}\cdot x} \sigma_z(x,y)|_\Gamma\,\mathrm{d}y\right)\mathrm{d}x$$

$$= -6\cdot\sqrt{3}\int_0^a \left(\sigma_z(x,y)|_\Gamma \cdot x\right)\mathrm{d}x \qquad (119$$

By substituting the radius a in Eq. (11) with the help of the obvious equation $a = |\delta|/\mathrm{tg}(\alpha)$ we can obtain a functional dependence between force P and depth of indentation δ (Fig. 4). From Eq. (11) we can obtain an approximate equality for penetrations $|\delta| > 0.03 \cdot h$:

$$P \approx \sqrt{3}\cdot\frac{E_y}{h\cdot\mathrm{tg}(\alpha)^2}\left(|\delta|^3 - 3\cdot\Omega\cdot\sigma_y\cdot h\cdot|\delta|^2\right) \tag{12}$$

5. Conclusions

A simple model of elastic–plastic penetration of a Berkovich pyramid into a multilayer coating was developed in this paper. Winkler's foundation was generalized to the case of elastic–plastic deformation of a multilayered coating. The deformation of a sublayer was modeled by Prandtl's bilinear approximation of the stress–strain curve. The homogenized coefficients (Eq. (5)) of the elastic and plastic properties of the layer set were found. An analytical solution of the problem was found.

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The depth of a pyramid penetration was obtained for different angles of the tip.

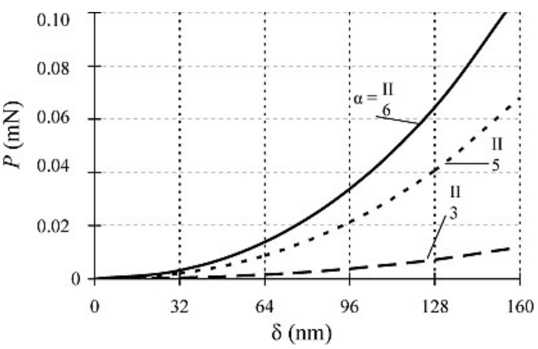

Fig. 4. Functional dependence between force and depth of indentation for different angles α $h_1 = 10$ nm, $h_2 = 50$ nm, $h_3 = 70$ nm, $h_4 = 30$ nm, $h_5 = 80$ nm; $E_1 = 200$ GPa, $E_2 = 100$ GPa, $E_3 = 150$ GPa, $E_4 = 80$ GPa, $E_5 = 240$ GPa; $\sigma_{y,1} = 0.24$ GPa, $\sigma_{y,2} = 0.15$ GPa, $\sigma_{y,3} = 0.3$ GPa, $\sigma_{y,4} = 0.24$ GPa, $\sigma_{y,5} = 0.08$ GPa, $E_{\mathrm{yield},1} = 4$ GPa, $E_{\mathrm{yield},2} = 0.15$ GPa, $E_{\mathrm{yield},3} = 1$ GPa, $E_{\mathrm{yield},4} = 0.24$ GPa, $E_{\mathrm{yield},5} = 0.8$ GPa. The indices 1, …, 5 are the number of layers in direction of increasing coating depth.

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