CONSISTENCY AND ASYMPROTIC DISTRIBUTION OF PARAMETER ESTIMATORS IN ARCH/GARCH MODELS WITH REGULARLY VARYING ERRORS

L.H. Son, N.N. Troush

Belarusian State Uniniversity, Department of Probability and Mathematical Statistic,
4 Nezavisimosti str., 220050 Minsk, Belarus
lhsondhv@gmail.com, troushnn@bsu.by

The consistency and asymptotic normality of patemeter estimators of ARCH/GARCH models are estabished when the errors have finite moment order 4^{th}, and the successful applications of ARCH/GARCH models were confined to the case of normal errors. When the errors have heavy-tail, P. Hall and Q. Yao showed that, the conventional quasi-miximum likelihood estimator suffers from complex limit distributions and slow convergence rates. In this paper, we survey the least absolute deviations estimation and modificative maximum likelihood estimation for ARCH and GARCH models with heavy-tailed errors. We established the consistency and asymptotic

distribution of parameter estimators for these models with errors, whose squares have regularly varying tail probabilities with the exponent α, $\alpha > 1$. Assume that, the GARCH model with orders $p \geqslant 1$ and $q \geqslant 0$ is given by:

$$y_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \text{ for } t \in Z,$$

where the parameters $\omega > 0$, $\alpha_i \geqslant 0$, and $\beta_j \geqslant 0$ are unknown parameters, $\{\epsilon_t\}$ is independent and identically distribution sequence, ϵ_1 is regularly varying with index α, $\alpha > 1$. Consider the following objective functions

$$L_n^{(1)}(\theta) = \frac{1}{n} \sum_{t=v+1}^{n} \left| y_t^2/\sigma_t(\theta)^2 - 1 \right|, \quad L_n^{(2)}(\theta) = \frac{1}{n} \sum_{t=v+1}^{n} \left| \ln\{y_t^2\} - \ln\{\sigma_t(\theta)^2\} \right|,$$

where $v = \min\{p, q\}$,

$$L_n^{(3)}(\theta) = \frac{1}{n} \sum_{t=1}^{n} l_t(\theta), \quad l_t(\theta) = -\left[\ln \sigma_t(\theta)^2 + \frac{1}{k} \left(\frac{y_t^2}{\sigma_t(\theta)^2} \right)^k \right],$$

where $\sigma_t(\theta)^2 = \omega + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}(\theta)^2$, $k > 0$, $\theta = (\omega, \alpha_1, \ldots, \alpha_p, \beta_1, \ldots, \beta_q)$.

Put

$$\widehat{\theta}_n^{(i)} = \arg\max_{\theta} L_n^{(i)}(\theta), \quad i = 1, 2, 3.$$

We showed that, for $i = 1, 2, 3$, the estimators $\widehat{\theta}_n^{(i)}$, are always consistence, while the asymptotic distribution of estimators $\widehat{\theta}_n^{(1)}$ and $\widehat{\theta}_n^{(2)}$ are always normality regardless the heavy-tailed property of the errors, and the asymptotic distribution of estimators $\widehat{\theta}_n^{(3)}$ is normality when $\alpha/k \geqslant 2$, but is α/k-stable when $1 < \alpha/k < 2$.