TAIL INDEX ESTIMATION FOR ARCH(1) PROCESS WITH HEAVY-TAIDED ERRORS

L.H. Son, I.I. Komarov

Belarusian State Uniniversity, Department of Probability theory and Mathematical Statistic,
4 Nezavisimosti str., 220050 Minsk, Belarus
`lhsondhv@gmail.com`, `komorov@brsu.best.by`

It is well known that, the class of processes ARCH/GARCH have heavy-tailed distribution, even their errors have light tail. This is one of the original motivations for the ARCH/GARCH processes, that they are widely used for modeling financial returns. One of popular estimator methods for estimating of tail thickness parameter for independent, identical distributional sequence is Hill's estimator, while the class of ARCH/GARCH processes cannot be successfully approximated by m-dependent random variables. Under general conditions such that the existence of a Lebesgue density of the errors, ARCH/GARCH processes are only ϕ-mixing. In this paper, using arguments in [2], we consider the estimation of tail thickness parameter of ARCH(1) process with errors, whose squares have α-stable distribution, $0 < \alpha < 2$.

Assume that, the ARCH(1) process is given by:

$$y_t = \sigma_t \epsilon_t, \quad \sigma_t^2 = \omega_0 + \alpha_0 y_{t-1}^2, \ \text{for} \ t \in Z,$$

where the parameters $\omega_0 > 0$, $\alpha_0 > 0$, $\{\epsilon_t^2\}$ is independent and identically distribution sequence of α-stable random variable, $0 < \alpha < 2$. In this case, this class of ARCH(1) process it satisfies the conditions of theorem 1 in [1], which ensure the heavy-tailed property of y_t and σ_t. i. e., there exist $k > 0$ and positive constants C_0 and C_1, such that

$$P(|y_t| > x) \sim C_0 x^{-k} \text{ and } P(|\sigma_t| > x) \sim C_1 x^{-k}, \text{ as } x \to \infty, \quad t \in Z.$$

Consider the Hill's estimator

$$H_{k,n}^y = \frac{1}{k} \sum_{i=1}^{k} \ln \frac{y_{(i)}}{y_{(k+1)}},$$

where $y_{(i)}$ is the i-th largest value of $y_1, y_2, \ldots, y_n$, for $1 \leqslant i \leqslant n$. We proved that, $H_{k,n}^y$ is consistent estimation of $1/(k)$ in the sense that

$$H_{k,n}^y \to {}^P 1/(k),$$

provided $n \to \infty$, $k \to \infty$ and $n/k \to \infty$.

Morever, the estimation of index k can be got by the absolute order k moment-formula of stable errors of the ARCH(1) process. Therefore, our result yields not only an estimator for the tail thickness parameters k of ARCH(1) process, but also an estimator for the index of stability α of the errors.

References

1. *Davis R., Mikosch T.* Extreme Value Theory for GARCH Processes. In T.G. Andersen, R.A. Davis, J.-P. Kreiss, and T. Mikosch, editors, Handbook of Financial Time Series, New York, Springer, 2008.

2. *Resnick S. and Starica C.* Tail Index Estimation for Dependent Data. The Annals of Applied Probability, 1998. V. 8. No. 4. P. 1156-1183.