A NOTE ON THE SPIN ALGEBRA IN A FRACTIONAL SPACE

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In 1927, W. Pauli proposed to consider the electron in the magnetic field as a particle that has a spin, i.e. a moment of its own. To the quantum value, spin $s$, corresponds the operator of spin $\hat{s}$ whose components are expressed by Pauli matrices [1]:

$$\hat{s}_i = \frac{1}{2} \sigma_i, \quad \sigma_i \sigma_k = -\sigma_k \sigma_i, \quad \sigma_i^2 = 1.$$  

(1)

Independently of a concrete representation, the spin algebra meets the following commutation relations [2]:

$$[\hat{s}_i, \hat{s}_j] = i \varepsilon_{ijk} \hat{s}_k, \quad [\hat{s}_i, \hat{s}^2] = 0.$$  

(2)

What will be the changes the commutation relations (2) undergo in a space of non-whole dimension?

The author shows that in the case when the effective dimension of space is not integer and may acquire fractional values [3], the electron becomes a particle with the spin projection $s_z = 1/2^\alpha$, where $\alpha \in [0; 1]$ and commutation relations for spin operators of fractional power

$$[\hat{s}_i^\alpha, \hat{s}_j^\alpha] = \frac{i}{2(\alpha - 1)} \sin^4 \left(\frac{\pi \alpha}{2}\right) \varepsilon_{ijk} \hat{s}_k, \quad [\hat{s}_i^\alpha, \hat{s}^2] = 0.$$  

(3)

The above peculiarities of electrons in a space of fractional dimension may contribute to theory of solid body, theory of superconductivity, the quantum effect of Hall, theory of phase transitions.

References