CHARACTERIZATIONS OF ALGEBRAS OF RAPIDLY DECREASING GENERALIZED FUNCTIONS

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The Schwartz space $\mathcal{S}$ of rapidly decreasing functions on $\mathbb{R}^n$ and its generalizations have been characterized by many authors and differently, e.g. see [3] and [1]. The most popular characterization of $\mathcal{S}$ is the following, let

$$\mathcal{S}^* = \left\{ f \in \mathcal{C}^\infty : \forall \alpha \in \mathbb{Z}_+^n, \sup_{x \in \mathbb{R}^n} |\partial^\alpha f(x)| < \infty \right\}, \quad \mathcal{S}_* = \left\{ f \in \mathcal{C}^\infty : \forall \beta \in \mathbb{Z}_+^n, \sup_{x \in \mathbb{R}^n} \left| x^\beta f(x) \right| < \infty \right\},$$

then, inspired by the work of [3], the authors of [1] proved the following result: $\mathcal{S} = \mathcal{S}^* \cap \mathcal{S}_*$.

To built a Fourier analysis within the generalized functions of [2], the algebra of rapidly decreasing generalized functions on $\mathbb{R}^n$, denoted $\mathcal{G}_\mathcal{S}$, was first constructed in [4]. The algebra of regular rapidly decreasing generalized functions on $\mathbb{R}^n$, denoted $\mathcal{G}_\mathcal{S}^\infty$, is fundamental in the characterization of the local regularity of a Colombeau generalized function by its Fourier transform and also for developing a generalized microlocal analysis.

The aim of this work is to characterize the algebras $\mathcal{G}_\mathcal{S}$ and $\mathcal{G}_\mathcal{S}^\infty$ in the spirit of the characterization of the Schwartz space $\mathcal{S}$ done in [1]. In fact we do more, this characterization is given in the general context of the algebras $\mathcal{G}_\mathcal{S}^\mathcal{R}(\Omega)$ of $\mathcal{R}$-regular rapidly decreasing generalized functions on an open set Ω of $\mathbb{R}^n$.

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