BROWNIAN MOTION OVER NON-ARCHIMEDEAN AND ADELIC DOMAINS

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We consider complex-valued Gaussian stochastic processes over p-adic domains and over rational adelic domains. Recall that p-adic field is locally compact completion of rational field $\mathbf{Q}$, and the ring $\mathbf{A}$ of adeles is built of all completions of rationals including the real one.

The most simple process is the Brownian bridge on the ring of p-adic integers $\mathbf{Z}_p$ considered earlier by Bikulov and Volovich [1]. We define adelic Brownian motion on the quotient space $\mathbf{A}/\mathbf{Q}$ by sum over rationals

$$B(t) = \sum_{k \in \mathbf{Q}^\times} \frac{1 - \chi(kt)}{h(k)} \xi(k).$$

Here χ is a non-trivial character on $\mathbf{A}/\mathbf{Q}$, $\xi(k)$ are independent standard Gaussian variables, and $h(k)$ is the complexity of a rational number k defined for irreducible $k = a/b$ by $h(k) = ab$.

Than we look at different properties of such processes, such as sample paths' continuity.

Considering such processes allows us to construct positively definite distributions on adelic spaces on a regular basis.

References

1. Bikulov A.Kh., Volovich I.V., p-Adic Brownian motion // Izv. RAN. Ser. Math. 1997. V. 61, No. 3. P. 75—90 (in Russian).