p-ADIC SPLINES AND THE INTEGRAL OPERATOR IN THE SPACE OF NON-ARCHIMEDIAN FUNCTIONS

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Let p be a prime number, $\alpha \in \mathbb{Q}$, $\alpha > 1$, and $\mathbb{Q}_p$ the field of p-adic numbers. Denote by $\mathbb{K}$ the minimal field containing p^α and $\mathbb{Q}_p$. The p-adic norm extended on $\mathbb{K}$ from $\mathbb{Q}_p$ will be non-archimedean (see [1] for details). We approximate an indicator of a ball with respect to the norm of $C(\mathbb{Z}_p;\mathbb{K})$ by p-adic splines, which are introduced here as a $\mathbb{K}$-linear combinations of translates of $|x|_p^{-\alpha}$ (see [2]). On the other hand the splines are very close to integral sums in Volkenborn's sense of the integral operator of kernel $|x-y|_p^{-\alpha}$. We show that the splines tend uniformly to the operator image of some function. We derive this function and then it is not hard to find pre-image of any locally constant function given on $\mathbb{Z}_p$ by linearity.

In a talk we discuss the continuous differentiability of $|x|_p^{-\alpha}$, the construction of the p-adic splines, a sketch proof of that the splines tend to the indicator, and that the splines tend to the image. We will present a pre-image of the indicator.

References

1. *Schikhof W.H.* Ultrametric calculus. An introduction to p-adic analysis. Cambrige University Press, 1984.

2. *Radyna A., Sender A.* The representation of a characteristic function of a ball by Riesz — Volkenborn's potential// Vestnik Grodnenskogo universiteta. Ser. 2. Physics. Mathematics. Informatics. Technology. Economics. 2007. N 2(52). P. 22–28.