ON A NULL SPACE OF THE PROBLEM WITH NONLOCAL TWO-POINT CONDITION FOR FIRST ORDER IN TIME PDE

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In the domain $t \in (0;\, h)$, $x \in \mathbb{R}^s$, we consider the boundary value problem

$$\left[\frac{\partial}{\partial t} - a\left(\frac{\partial}{\partial x}\right)\right] U(t,x) = 0, \quad p\left(\frac{\partial}{\partial x}\right) U(0,x) + q\left(\frac{\partial}{\partial x}\right) U(h,x) = 0, \tag{1}$$

where $a\left(\frac{\partial}{\partial x}\right)$ is an arbitrary differential expression with constant complex coefficients, whose symbol is an entire analytical function $a(\nu)$, $h \in \mathbb{R}$, $h > 0$, $p\left(\frac{\partial}{\partial x}\right)$, $q\left(\frac{\partial}{\partial x}\right)$ are differential polynomials with complex coefficients.

By means of the Differential-symbol method [1, 2] we have found a way of constructing quasipolynomial solutions of problem (1). Those solutions are being constructed in the form

$$U(t,x) = g\left(\frac{\partial}{\partial \nu}\right)\{\exp\left[a(\nu)t + \nu \cdot x\right]\}, \tag{2}$$

where $g\left(\frac{\partial}{\partial \nu}\right)$ is a differential expression whose symbol is a quasipolynomial of a special form.

Let us show the result for the case of one spatial variable ($s = 1$). Consider the function $\eta(\nu) = p(\nu) + q(\nu)\exp\left[a(\nu)h\right]$ and the set of its zeros P . Let $p_\alpha \in P$ denote a multiplicity of the zero α. Consider also the class K_M of quasipolynomials of the form

$$f(x) = \sum_{j=1}^{m} \exp[\alpha_j x] Q_j(x),$$

where $Q_j(x)$ are polynomials with complex coefficients, $\alpha_j \in M \subseteq \mathbb{C}$, $x \in \mathbb{R}$, $m \in \mathbb{N}$.

Theorem 1. *Let $g(x)$ be quasipolynomial from K_P and have the form*

$$g(x) = \sum_{j=1}^{m} \exp[\alpha_j x] Q_j(x),$$

where $Q_j(x)$ are polynomials of degrees $n_j \leqslant p_{\alpha_j}$, $j = \overline{1,m}$, with complex coefficients. Then function (2) *is a solution of problem* (1).

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References

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