DISCRETE LIMIT THEOREMS FOR HURWITZ ZETA-FUNCTION

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The Hurwitz zeta-function $\zeta(s,\alpha)$, $s=\sigma+it$, with fixed parameter α, $0<\alpha\leqslant 1$, is defined by

$$\zeta(s,\alpha)=\sum_{m=0}^{\infty}\frac{1}{(m+\alpha)^s},\quad \sigma>1.$$

It has an analytic continuation to the whole complex plane $\mathbb{C}$, except for a simple pole at the point $s=1$ with residue 1. For $\alpha=1$, the function $\zeta(s,\alpha)$ reduces to the Riemann zeta-function.

In 2002–2007, continuous limit theorems in the sense of the weak convergence of the probability measures for the Hurwitz zeta-function with transcendental, rational or algebraic irrational parameter α by A. Laurinčikas, R. Garunkštis and J. Steuding were proved. In 2003, a discrete limit theorems for $\zeta(s,\alpha)$ with transcendental α by J. Ignatavičūtė were obtained [1].

In our talk, two probabilistic results for the Hurwitz zeta-function with algebraic irrational parameter α on the complex plane will be presented. Here we present one of them.

Let $h>0$ be a fixed number such that $\exp\{2\pi/h\}$ is rational. We suppose that there are algebraic irrational numbers α such that, for every collection $\underline{k}=(k_0,k_1,\dots)$, where only a finite number of integers k_m are distinct from zero, the product $\prod_{m=0}^{\infty}(m+\alpha)^{k_m}$ is irrational.

Define, for $\sigma>1/2$, on the probability space $(\Omega,\mathcal{B}(\Omega),m_H)$ the complex-valued random element $\zeta(\sigma,\alpha,\omega)$ by

$$\zeta(\sigma,\alpha,\omega)=\sum_{m=0}^{\infty}\frac{\omega(m)}{(m+\alpha)^\sigma}$$

(for the definition of $\omega(m)$, Ω and m_H, see, for example, [2]).

Theorem. *Suppose that* α *and* h *satisfy the above mentioned conditions. Let* $\sigma > 1/2$. *Then on* $(\mathbb{C}, \mathcal{B}(\mathbb{C}))$ *the probability measure*

$$\frac{1}{N+1}\#\{\zeta(\sigma + ilh, \alpha) \in A\}, \quad A \in \mathcal{B}(\mathbb{C}),$$

converges weakly to the distribution of the random element $\zeta(\sigma, \alpha, \omega)$ *as* $N \to \infty$.

The second result deals with a generalization of Theorem for a collection of Hurwitz zeta-functions with different parameters α. It can be found in [3].

References

1. *Ignatavičiūtė J.* Value-Distribution of the Lerch Zeta-Function. Discrete Version. Doctoral Thesis, Vilnius University, 2003.

2. *Kačinskaitė R., Laurinčikas A.* For the Hurwitz zeta-function with an algebraic irrational parameter // Ann. Univ. Sci. Budap. Rolando Eцtvцs, **29**, 2008. P. 25–38.

3. *Kačinskaitė R., Korsakienė D., Steuding R.* A joint discrete value distribution of Huwitz zeta-functions with algebraic irrational parameters. II // Šiauliai Math. Semin. **3**(11), 2008. P. 153–168.