THE KANONICAL NORMAL AFFINE STOLYAROV'S CONNECTION WITHOUT TORSION ON THE DISTRIBUTION OF PLANES

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Let's consider projective n-dimensional space P_n and moving frame in it $\{A, A_I\}$ $(I, \ldots = \overline{1, n})$ with derivative formulaes

$$dA = \theta A + \omega^I A_I, \quad dA_I = \theta A_I + \omega_I^J A_J + \omega_I A,$$

besides the structure forms ω^I, ω_J^I, ω_I of projective group, which is effectively operated in space P_n and satisfy to Cartan's equations

$$D\omega^I = \omega^J \wedge \omega_J^I, \quad D\omega_J^I = \omega_J^K \wedge \omega_K^I + \delta_J^I \omega_K \wedge \omega^K + \omega^J \wedge \omega^I, \quad D\omega_I = \omega_I^J \wedge \omega_J.$$

Let's consider m-distribution in P_n, i.e. n-parametrical family S_n of centered m-planes P_m^* in space P_n. Place the vertices A, A_i $(i,\ldots=\overline{1,m})$ into plane P_m^*, besides A - in its center. Then we write equations of the distribution S_n in the following manner

$$\omega_i^\alpha=\Lambda_{iJ}^\alpha\omega^J,\quad \Delta\Lambda_{iJ}^\alpha-\delta_J^\alpha\omega^i=\Lambda_{iJK}^\alpha\omega^K\quad (\alpha,\ldots=\overline{m+1,n}),$$

$$\Delta\Lambda_{iJ}^\alpha=d\Lambda_{iJ}^\alpha+\Lambda_{iJ}^\beta\omega_\beta^\alpha-\Lambda_{jJ}^\alpha\omega_\beta^j-\Lambda_{iK}^\alpha\omega_J^K.$$

Define normal affine Stolyarov's connection by means of forms $\widetilde{\omega}^\alpha=\omega^\alpha-L_I^\alpha\omega^I$, $\widetilde{\omega}_\beta^\alpha=\omega_\beta^\alpha-$ $-\Gamma_{\beta I}^\alpha\omega^I$, besides components of the connection's object $\Gamma=\{L_I^\alpha,\Gamma_{\beta I}^\alpha\}$ satisfy to differential equations

$$\Delta L_I^\alpha=L_{IJ}^\alpha\omega^J,\quad \Delta\Gamma_{\beta I}^\alpha-\Lambda_{iJ}^\alpha\omega_\beta^i-\delta_\beta^\alpha\omega_I-\delta_I^\alpha\omega_\beta=\Gamma_{\beta IJ}^\alpha\omega^J.$$

Condition $L_I^\alpha=0$ allocates canonical connection $\Gamma^0=\{0,\Gamma_{\beta I}^\alpha\}$, where components of its torsion tensor are expressed by formulaes

$$T_{ij}^\alpha=\Lambda_{[ij]}^\alpha,\quad T_{\beta i}^\alpha=\frac{1}{2}(\Gamma_{\beta i}^\alpha-\Lambda_{i\beta}^\alpha),\quad T_{\beta\gamma}^\alpha=\Gamma_{[\beta\gamma]}^\alpha.$$

Theorem 1. *The canonical normal affine Stolyarov's connection without torsion on the distribution is characterised by sequence of properties: the distribution is holonomic* $(\Lambda_{ij}^\alpha=\Lambda_{ji}^\alpha)$, *linear connection* $\Gamma_{\beta I}^\alpha$ *is half-intrinsic* $(\Gamma_{\beta i}^\alpha=\Lambda_{i\beta}^\alpha)$ *and the connection* $\Gamma_{\beta I}^\alpha$ *is symmetrical* $(\Gamma_{\beta\gamma}^\alpha=$ $=\Gamma_{\gamma\beta}^\alpha)$.