ON QUASI-$\mathcal{F}$-GROUPS

Wenbin Guo [1], **A.N. Skiba** [2]

[1] Department of Mathematics, Xuzhou Normal University,
221116 Xuzhou, P.R. China
wbguo@xznu.edu.cn

[2] Department of Mathematics, Francisk Skorina Gomel State University,
Sovetskaya 104, 246019 Gomel, Belarus
alexander.skiba49@gmail.com

Let $\mathcal{F}$ be a class of groups. A chief factor H/K of a group G is called $\mathcal{F}$*-central* provided $[H/K]\operatorname{Aut}_G(H/K) \in \mathcal{F}$. Otherwise, it is called $\mathcal{F}$*-eccentric.*

Definition 1. *Let $\mathcal{F}$ be a class of groups and G a group. We say that G is a quasi-$\mathcal{F}$-group if for every $\mathcal{F}$-eccentric chief factor H/K of G, every automorphism of H/K induced by an element of G is inner.*

We discuss some aspects of a general theory of finite quasi-$\mathcal{F}$-groups [1] and consider some its applications in studying of finite non-nilpotent groups. In particular, characterizations of finite quasisoluble and quasisupersoluble groups are given. Besides, a new characterization of finite quasinilpotent groups is obtained.

References

1. *Wenbin Guo, Skiba A.N.* On finite quasi-$\mathcal{F}$-groups // Comm. Algebra (in press).

RECOGNITION BY SPECTRUM AMONG COVERS OF THE SIMPLE GROUPS $\mathrm{PSL}_n(q)$

A.V. Zavarnitsine

Sobolev Institute of Mathematics,
4 Koptyug av., 630090 Novosibirsk, Russia
zav@math.nsc.ru

If a group H is a homomorphic image of a finite group G, we say that G is a *cover* for H. The talk will be devoted to solving [1, Problem 14.60]:

Problem 1. Suppose that G is a proper cover for a finite simple group $L = \mathrm{PSL}_n(q)$, $n \geqslant 3$. Is it true that G contains an element whose order is distinct from the order of every element of L?

There is a direct relation between this problem and the problem of recognizing finite groups by spectrum. Recall that the *spectrum* $\omega(H)$ of a finite group H is the set of its element orders. We say that H is *recognizable (by spectrum) among its covers* if, for every finite group G covering H, the equality of the spectra $\omega(G) = \omega(H)$ implies the isomorphism $G \cong H$. Thus, Problem 1 asks if every simple group $\mathrm{PSL}_n(q)$, $n \geqslant 3$, is recognizable among its covers.

It can be shown that the analysis of Problem 1 may be reduced to the case where the cover G is a natural semidirect product $W \rtimes L$, with W being an elementary abelian p-group, p the defining characteristic for $L = \mathrm{PSL}_n(q)$, and the action of L on W being faithful and absolutely irreducible. This leads naturally to the study of the representations for L in the defining characteristic.

Using the properties of weights of the irreducible modules for algebraic groups of type A_l, we prove the following theorem:

Theorem 1. *Let* $L = \mathrm{PSL}_n(q)$ *be a simple group. If either* $n \neq 4$, *or* q *is prime, or* q *is even, then* L *is recognizable by spectrum among its covers.*

It turns out that the groups $\mathrm{PSL}_4(q)$, with q odd and nonprime, exhibit a certain exceptional behavior when acting in the defining characteristic. This can be seen from the following assertion.

Theorem 2. *The group* $L = \mathrm{PSL}_4(13^{24})$ *has an absolutely irreducible* 96*-dimensional module* W *over a field of characteristic* 13 *such that* $\omega(W \leftthreetimes L) = \omega(L)$. *In particular,* L *is not recognizable by spectrum among its covers.*

Therefore, the question in Problem 1 is answered in the affirmative if $n \neq 4$ and, in general, in the negative if $n = 4$.

Acknowledgement. This research was partially supported by the Russian foundation for basic research (projects 06-01-39001 and 08-01-00322), the council of the President grants (project NSh-344.2008.1), the foundation for support of the national science (grant of 2008), and the Siberian branch of the Russian academy of sciences (Integration project 2006.1.2).

References

1. Unsolved problems in group theory, *The Kourovka notebook*, 14th ed., Sobolev Inst. Mat (Novosibirsk), 1999.