1) $J_\phi(z) = \mathbb{N}_1^p$;
2) $p \equiv 2(\mod 3)$, ω *or* $\omega^* = \frac{p+1}{3}\omega_1$, $J_\phi(z) = \mathbb{N}_3^{p-4} \cup \{1, p-1, p\}$;
3) ω *or* $\omega^* = a_k\omega_k + \ldots + a_l\omega_l$, $k \leqslant l < n-2$, $a_k a_l \neq 0$. a_l *or* $a_{l-1} + a_l = p-1$, $a_j + a_{j+1} + +a_{j+2} \neq 0$ *for* $k < j < l$, a_k *or* $a_{k+1} + a_k = p-1$ *for* $k > 3$, $\mathbb{N}_1^p \setminus \{2, p-2\} \subset J_\phi(z)$;
4) ω *or* $\omega^* = a_1\omega_1$, $a_1 > \frac{p+1}{3}$, $\mathbb{N}_1^p \setminus \{2, p-2\} \subset J_\phi(z)$;
5) $p \equiv 1(\mod 3)$, ω *or* $\omega^* = \frac{p-4}{3}\omega_1 + \omega_j$, $1 < j < n$, $\mathbb{N}_1^p \setminus \{p-1\} \subset J_\phi(z)$.

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ON $\mathbb{Q}$-CONIC BUNDLES

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The talk is based on joint works with Shigefumi Mori [1–3].

A $\mathbb{Q}$-conic bundle is a proper morphism from a threefold with only terminal singularities to a normal surface such that fibers are connected and the anti-canonical divisor is relatively ample. We study the structure of $\mathbb{Q}$-conic bundles near their singular fibers. The complete classification of $\mathbb{Q}$-conic bundles is obtained under the additional assumption that the base surface is singular. In particular, we show that the base surface of every Q-conic bundle has only Du Val singularities of type A (a positive solution of a conjecture by Iskovskikh). Under бertain additional assumptions we prove M. Reid's general elephant conjecture.

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EXISTENCE AND CONJUGACY OF HALL SUBGROUPS IN FINITE GROUPS

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The term "group" always means a finite group. In what follows π is a set of primes, π' is its complement in the set of all primes, $\pi(n)$ is the set of all prime divisors of a rational integer n. A positive integer n is called a π-number if all its prime divisors are in π. For a group G we set $\pi(G)$ to be equal to $\pi(|G|)$. A subgroup H of G is called a π-Hall subgroup if $\pi(H) \subseteq \pi$ and $\pi(|G : H|) \subseteq \pi'$.

According to P. Hall, we say that G *satisfies* E_π (or briefly $G \in E_\pi 0$, if G contains a π-Hall subgroup. If $G \in E_\pi$ and every two π-Hall subgroups are conjugate, we say that G *satisfies* C_π ($G \in C_\pi 0$. If $G \in C_\pi$ and each π-subgroup of G is included in a π-Hall subgroup of G, we say that G *satisfies* D_π ($G \in D_\pi 0$. Let A, B, H be subgroups of G such that $B \trianglelefteq A$ and

$H \leqslant G$. Then $N_H(A/B) = N_H(A) \cap N_H(B)$ is the normalizer of A/B. If $x \in N_H(A/B)$, then x induces an automorphism of A/B by $Ba \mapsto Bx^{-1}ax$. Thus there exists a homomorphism $N_H(A/B) \to \mathrm{Aut}(A/B)$. The image of $N_H(A/B)$ under this homomorphism is denoted by $\mathrm{Aut}_H(A/B)$.

In 2006 the authors proved that a group G with a normal subgroup A satisfies D_π if and only if G/A and A satisfy D_π. So G satisfies D_π if and only if every its composition factor satisfies D_π. We investigate whether $G \in E_\pi$ or $G \in C_\pi$, in terms of the normal structure of G or the composition structure of G.

It is known that if $A \trianglelefteq G$ and $G \in E_\pi$, then G/A and A satisfy E_π. There are examples showing that the reverse statement is not true. In 1986 F. Gross proved that if $1 = G_0 < < G_1 < \ldots < G_k = G$ is a composition series of G which is a refinement of a chief series, and $\mathrm{Aut}_G(G_i/G_{i-1}) \in E_\pi$ for all i, then $G \in E_\pi$. We prove the opposite statement, so the following theorem holds.

Теорема 1. *Let $1 = G_0 < G_1 < \ldots < G_k = G$ be a composition series of G, which is a refinement of a chief series. Then $G \in E_\pi$ if and only if $\mathrm{Aut}_G(G_i/G_{i-1}) \in E_\pi$ for all $i = 1, \ldots, k$*

S.A. Chunikhin in 1952 proved that a group G with a normal subgroup A satisfies C_π if both G/A and A satisfy C_π. It follows from Theorem 1 that $G \in C_\pi$ implies $G/A \in C_\pi$. Known examples show that if $G \in C_\pi$, then A may fail to satisfy C_π. We obtain the following theorem in this direction.

Theorem 1. *Let A be a normal subgroup of G. Then $G \in C_\pi$ if and only if $G/A \in C_\pi$ and, for a π-Hall subgroup K/A of G/A, we have $K \in C_\pi$.*

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WEYL SUBMODULES IN RESTRICTIONS OF SIMPLE MODULES

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Let G be a universal Chevalley group over an algebraically closed field $\mathbb{F}$, $L(\omega)$ be the simple rational G-module with highest weight ω and $v^+{}_\omega$ be a highest weight vector of $L(\omega)$. Let $G^{(q)}$ denote the subgroup of G generated by the root elements $x_{\pm\alpha}(t)$, where α is a simple root distinct from a fixed terminal root α_q. We consider the $G^{(q)}$-submodule V of $L(\omega)$ generated by $X_{-\alpha_q,k}v^+{}_\omega$, where $X_{-\alpha_q,k} = X^k_{-\alpha_q}/k! \otimes 1_\mathbb{F}$ and $k < p = \mathrm{char}\,\mathbb{F}$, is considered. For $G = A_\ell(\mathbb{F})$, we prove that V is isomorphic to a Weyl module if and only if $V \neq 0$ and certain Hom-spaces between Weyl modules equal zero. This allows us to embed Weyl modules into $L(\omega){\downarrow}_{G^{(q)}}$ under exact combinatorial conditions.