attempts to treat a triangulated category, or maybe a DG or an A_∞-category, as a geometric object with all that it entails. The desire to do so comes originally from physics, where interesting A_∞-categories such as the Fukaya category appear in the context of topological field theories.

Mathematically, main motivation comes from the usual algebraic geometry, where one attempts to extract information about an algebraic variety X from its derived category $\mathcal{D}^b(X)$ of coherent sheaves. When passing from X to $\mathcal{D}^b(X)$, some information is certainly lost — different varieties can have equivalent derived categories. What is surprising is how much can be recovered. Here are some the invariants of X which are completely determined by $\mathcal{D}^b(X)$ (or rather, its A_∞-version):

1) algebraic K-theory $K^{\cdot}(X)$,

2) differential forms Ω^i_X — these correspond to Hochschild homology classes of $\mathcal{D}^b(X)$,

3) de Rham cohomology $H^{\cdot}_{DR}(X)$ — this corresponds to periodic cyclic homology of $\mathcal{D}^b(X)$.

It is expected that much more is true: loosely speaking, all the "motivic"structures which exits on the de Rham cohomology $H^{\cdot}_{DR}$ should also exist on the periodic cyclic homology of a nice enough A_∞-category $\mathcal{C}$.

In the talk, I will present a recent discovery in this direction: it turns out that for a nice enough A_∞-category $\mathcal{C}$ defined over a finite field k, or over the Witt vectors ring $W(k)$, the periodic cyclic homology $HP_{\cdot}(\mathcal{C})$ carries a natural action of the Frobenius map, and moreover, has a structure of a "filtered Diedonné module" of Fontaine-Lafaille — the p-adic analog of a mixed Hodge structure. This allows, among other things, to prove a Hodge-to-de Rham degeneration result for $\mathcal{C}$ using the well-known method of Deligne and Illusie. I will also discuss the relation with the notion of syntomic cohomology, and present a p-adic non-commutative version of the Beilinson conjecture and the regulator map.

QUANTUM LEIBNIZ ALGEBRAS AND THEIR CONFORMAL REPRESENTATIONS

P.S. Kolesnikov

Sobolev Institute of Mathematics,
4 Acad. Koptyug ave., 630090 Novosibirsk, Russia
pavelsk@math.nsc.ru

Quantum Lie algebras introduced by S. Woronowicz [1] are related to the axiomatic approach to the first order differential calculus (FODC) over a Hopf algebra. By definition, a quantum Lie algebra $\mathfrak{g}$ is a linear space equipped with a braiding $\sigma : \mathfrak{g}\otimes\mathfrak{g} \to \mathfrak{g}\otimes\mathfrak{g}$ and with a linear bracket product $\mu = [\cdot,\cdot] : \mathfrak{g}\otimes\mathfrak{g}\to\mathfrak{g}$ satisfying the following axioms:

$$\mu(\mathrm{id}\otimes\mu) = \mu(\mu\otimes\mathrm{id}) + \mu(\mathrm{id}\otimes\mu)(\sigma\otimes\mathrm{id}), \tag{1}$$

$$\sigma(\mathrm{id}\otimes\mu) - (\mu\otimes\mathrm{id})\sigma_{23}\sigma_{12} = 0, \tag{2}$$

$$\sigma(\mu\otimes\mathrm{id}) - (\mathrm{id}\otimes\mu)\sigma_{12}\sigma_{23} = (\mu\otimes\mathrm{id})(\mathrm{id}\otimes\sigma) - \sigma(\mathrm{id}\otimes\mu)(\sigma\otimes\mathrm{id}), \tag{3}$$

$$\mathrm{Ker}\,(\mathrm{id}-\sigma) \subseteq \mathrm{Ker}\,\mu, \tag{4}$$

If σ is the ordinary flip of tensor factors then (1) coincides with the Jacobi identity, (2) and (3) trivially hold, (4) represents skew-symmetry of μ.

Therefore, it is natural to call a system $(\mathfrak{g},\sigma,\mu)$ satisfying (1)–(3) to be a non-commutative analogue of a quantum Lie algebra, i.e., a *quantum Leibniz algebra*.

On the other hand, conformal algebras were introduced by V. Kac [2] as a tool of studying the structure and representation theory of vertex operator algebras. One of the most important examples of conformal algebras is provided by conformal endomorphisms. Let us state the corresponding notion in a little bit more general context.

Let G be a linear algebraic group over an algebraically closed field $\Bbbk$ and let H be its coordinate Hopf algebra. A G-*conformal endomorphism* of a left H-module M is a map $a : G \to \mathrm{End}_{\Bbbk} M$ such that

- for every $u \in M$ the map $\gamma \mapsto a(\gamma)u$ is a regular function from G to M;
- $a(\gamma)h = L_\gamma h a(\gamma)$ for $h \in H$, $\gamma \in G$, where $L_\gamma h : x \mapsto h(\gamma x)$, $x \in G$.

In the case of $G = \mathbb{A}^1 \simeq (\Bbbk, +)$, $\mathrm{char}\,\Bbbk = 0$, this notion corresponds to the one of [2]. Denote by $\mathrm{Cend}\,M$ the space of all G-conformal endomorphisms of an H-module M.

Definition 1. *A G-conformal representation of a quantum Leibniz algebra $\mathfrak{g}$ on an H-module M is a linear map $\rho : \mathfrak{g} \to \mathrm{Cend}\,\mathfrak{M}$ such that*

$$\rho(a)(e)(\rho(b)(\gamma)v) - \sum_i \rho(b_i)(\gamma)(\rho(a_i)(e)v) = \rho([a, b])(\gamma)v,$$

where $\sum_i b_i \otimes a_i = \sigma(a \otimes b)$, $a, b \in \mathfrak{g}$, $\gamma \in G$, $v \in M$, e is the unit of G.

Theorem 1. *If G is a linear algebraic group such that H contains a primitive element then a (finite-dimensional) quantum Leibniz algebra has a faithful G-conformal representation on an appropriate (finitely generated) H-module M.*

For example, every finite-dimensional quantum Leibniz algebra can be embedded into the conformal algebra (over $G = \mathbb{A}^1$) $\mathrm{Cend}\,M$, where M is a finitely generated free $\Bbbk[T]$-module.

References

1. *Woronowicz S.L.* Differential calculus on compact matrix pseudogroups (quantum groups) // Comm. Math. Phys. 1989. V. 122. P. 125–170.
2. *Kac V.G.* Vertex algebras for beginners. Second edition. Providence, RI: AMS, 1998. (University Lecture Series, vol. 10).

PERIODIC GROUPS WITH PRESCRIBED ELEMENT ORDERS

V.D. Mazurov

Sobolev Institute of Mathematics, the Siberian Branch of Russian Academy of Sciences,
Novosibirsk, 630090, Russia
mazurov@math.nsc.ru

For a periodic group G, denote by $\omega(G)$ the *spectrum*, i.e. the set of element orders, of G. It is obvious that $\omega(G)$ is finite if and only if G is of finite exponent. Thus, a group with finite spectrum is not necessarily a locally finite group.

The talk contains a survey of known spectra which ensure the local finiteness of corresponding groups. The following recent results are typical.

Theorem 1. *Let $\omega(G) = \{1, 2, 3, 5, 6\}$. Then G is locally finite.*

Theorem 2. *Let $\omega(G) = \{1, 2, 3, 4, 8\}$. Then G is locally finite.*