NON-COMMUTATIVE CURVES AND TILTING

I.I. Burban[1], **Y.A. Drozd**[2]

[1] Mathematisches Institut Universität Bonn,
Beringstr. 1, D-53115 Bonn, Germany
burban@math.uni-bonn.de

[2] Institute of Mathematics, National Academy of Sciences of Ukraine,
Tereschenkivska str. 1, 01601 Kyiv, Ukraine
drozd@imath.kiev.ua

A *non-commutative curve* is a pair $(X, \mathcal{A}_X)$ i, where X is a (usual) algebraic curve and $\mathcal{A}_X$ is a sheaf of $\mathcal{O}_X$-algebras coherent as a sheaf of $\mathcal{O}_X$-modules. We denote by $\mathrm{Coh}\mathcal{A}_X$ the category of coherent sheaves of $\mathcal{A}_X$-modules.

Let X be a (usual) projective curve over a field $\mathbf{k}$ such that all its components are rational and all singular points are simple double points, S be the set of singular points of X, $\pi : Y \to$ char†∈∞ X be its normalization, $Y_1, Y_2, \dots, Y_s$ be the irreducible components of Y, all identified with the projective line $\mathbb{P}^1$, $\{x', x''\} = \pi^{-1}(x)$ for $x \in S$, $\mathcal{O}_X^* = \pi_* \mathcal{O}_Y$, $\mathcal{M} = \mathcal{O}_X \oplus \mathcal{O}_X^*$ and $\mathcal{A}_X = \mathcal{E}\backslash_{\mathcal{O}_X}(\mathcal{M})$.

Theorem 1. 1. $\mathrm{gl.dim.}\mathcal{A}_X = 2$, *i.e.* $\mathrm{Ext}^2_{\mathcal{A}_X}$ *vanishes.*

2. $\mathcal{T} = \mathcal{O}_X^* \oplus \mathcal{O}_X^*(1) \oplus \bigoplus_{x \in S} \mathbf{k}(x)[-1]$, *where* $\mathbf{k}(x)$ *is the resudue field of the point* x, *is a tilting object in the derived category* $D^b(\mathrm{Coh}\mathcal{A}_X)$, *so* $D^b(\mathrm{Coh}\mathcal{A}_X) \simeq D^b(\mathbf{A}\text{-mod})$, *where* $\mathbf{A}_X = \mathrm{End}_{D^b(\mathrm{Coh}\mathcal{A}_X)}(\mathcal{T})$.

3. $\mathbf{A}_X \simeq \mathbf{k}\Gamma/I$, *where:*

Γ *is the quiver (oriented graph) with the set of vertices* $S \cup \{y_i, y_i' \mid 1 \leqslant i \leqslant s\}$ *and the set of arrows* $\{a_Z, b_Z \mid Z \in C\} \cup \{c_x', c_x'' \mid x \in S\}$ *such that* $a_i, b_i : y_i \to y_i'$, $c_x' : x \to y_j$ *and* $c_x'' : x_k \to y$ *if* $x' \in Y_j$, $x'' \in Y_k$;

I *is the ideal of the path algebra* $\mathbf{k}\Gamma$ *generated by the elements* $(\eta_i' a_j - \xi_i' b_j)c_x' = 0$ *and* $(\eta_i'' a_k - \xi_i'' b_k)c_x''$, *where* $(\xi_i' : \eta_i')$ *and* $(\xi_i'' : \eta_i'')$ *are, respectively, the homogeneous coordinates of* x_i' *on* Y_j *and of* x_i'' *on* Y_k.

Note that the algebra $\mathbf{A}_X$ first appeared in [1], where the authors showed that its representation type coincides with that of the curve X and asked for an *a priori* explanation of this phenomenon. Theorem 1 gives such an explanation, since the category $\mathrm{Coh}\mathcal{O}_X$ naturally embeds into $\mathrm{Coh}\mathcal{A}_X$.

We also present generalizations of Theorem 1 for other classes of curves (both usual and non-commutative).

This work was supported by Mathematische Institut Oberwolfach (Program RiP 2008). The second author was also supported by INTAS Grant 06-1000017-9093.

References

1. *Drozd Y.A., Greuel G.-M.* Tame and wild projective curves and classification of vector bundles // J. Algebra. 2001. V. 246. No. 1. P. 1–54.

MOTIVIC STRUCTURES IN NON-COMMUTATIVE GEOMETRY

D. Kaledin

Steklov Math Institute, Moscow, Russia
kaledin@mi.ras.ru

"Non-commutative geometry" is a term with many meanings. I am following the recent usage, which equates non-commutative geometry with "geometry of triangulated categories" — one