METHOD OF SINGULAR WAVELET: DATA APPROXIMATION AND SMOOTHING ACCORDING TO PROCESS EFFECTIVENESS

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Abstract

This work examines the universal algorithm of function approximation by method of singular wavelets, which allows to solve a problem of function smoothing and interpolation (quasi-interpolation) given on non-uniform mesh at $\mathbf{R}^n$.

1 Mathematical model, short description of results

The mathematical model is presented as follows: $Y = f(x_1, x_2, \dots, x_m, a_1, a_2, \dots, a_n)$, where Y is an indicator being evaluated, f is any analytical function, $x_1, x_2, \dots, x_m$ are factors, $a_1, a_2, \dots, a_n$ are parameters of the function being determined.

Wavelet transforms theory [1] is developed within the signal theory. The following expression is called the integral wavelet transform of the function $f(t)$: $W_\psi(f)(b,a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t)\psi\left(\frac{t-b}{a}\right) dt$. Function $\psi(t)$ in this expression is called "wavelet" and it should satisfy the condition

$$\int_{-\infty}^{+\infty} \psi(t)\, dt = 0. \tag{1}$$

Functions that may be wavelets should satisfy the admissibility condition (1). This narrows the class of functions which may be used as wavelets.

On the other part, the method of smooth delta function (Rosenblatt, 1956; Parzen, 1962) is widely used in estimation of distribution densities and regression relationships [2]. The proposed approach assumes that analytic link function is represented by wavelet transform, where smooth delta function are used as wavelets [2].

Let's introduce now the interpretation of singular wavelet transform for $\psi \in L^2(\mathbf{R})$. Continuous singular wavelet transform is determined as follows:

$$W_\psi\big(f - f(b)\big)(b,a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} \big(f(t) - f(b)\big)\psi\left(\frac{t-b}{a}\right) dt.$$

In particular case, when the admissibility condition (1) is met for the function $\psi(t)$, singular integral wavelet transform coincides with an ordinary integral wavelet transform:

$$W_\psi\big(f - f(b)\big)(b,a) = W_\psi(f)(b,a).$$

One may obtain formula for inversion of singular integral wavelet-transform [3]:

$$f(t) = \frac{1}{C_\psi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} W_\psi\big(f - f(b)\big)(b,a) \left(\frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right)\right) \frac{db\, da}{a^2}, \tag{2}$$

where $C_\psi = \int_{-\infty}^{+\infty} \omega^{-1} \hat{\psi}(\omega) \left(\hat{\psi}(\omega) - \hat{\psi}(0)\right) d\omega < \infty$, $\hat{\psi}(\omega)$ Fourier transform function $\psi(t)$. In discrete case it is suggested to use estimation:

$$f(t) = \sum_{k=0}^{K} \left(\sum_{m=1}^{M} U^k(b_m)\psi\left(\frac{t-b_m}{a_k}\right)\right) / \left(\sum_{m=1}^{M} \psi\left(\frac{t-b_m}{a_k}\right)\right), \qquad (3)$$

where k - approximation number, K - maximal order of approximation, M number of approximation nodes (points of factor space), $a_k = 2^{-k}C$, $C = \text{const}$, b_m interpolation nodes $m = 1, 2, \ldots, M$, $U^k(b_m)$ discrete values of generalized wavelet-transform in interpolation nodes.

Values $U^k(b_m)$, $n \in \{1, 2, \ldots, M\}$, may be calculated by using recurrent formulae:

$$U^0(b_n) = f(b_n),$$

$$U^{k+1}(b_n) = \left(\sum_{m=1}^{M} \left(U^k(b_n) - U^k(b_m)\right)\psi\left(\frac{b_n - b_m}{a_k}\right)\right) / \left(\sum_{m=1}^{M} \psi\left(\frac{b_n - b_m}{a_k}\right)\right).$$

In particular case, when $K = 0$, wavelet series coincides with the estimation (Nadaraya, 1964; Watson, 1964) [3].

In Fig. 1 a fragment of quasi-interpolation of single-factor link function $y = | x |$ is shown (the solid line plots the approximation by discrete wavelet-series, dotted lines are the Nadaraya-Watson estimation, dots indicate the function $y = | x |$, circles are interpolation nodes).

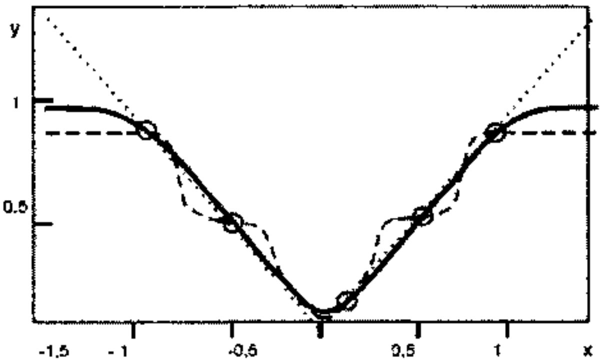

Figure 1: Approximation of theoretical link function $y = | x |$, using different methods for nonuniform distribution of interpolation nodes

Multivariate case results through natural generalization of one-dimensional case [3].

In Fig. 2 a fragment of estimation of function $y = (x_1 - 0,5)^2 + (x_2 - 0,5)^2$ (paraboloid) is shown (1 - model function $y = (x_1 - 0,5)^2 + (x_2 - 0,5)^2$; 2 - approximation of function on interpolation nodes; a - - for number of interpolation nodes $N = 7$; b - - for number of interpolation nodes $N = 25$; c -- distribution of $N = 7$; d - - distribution of interpolation nodes $N = 25$).

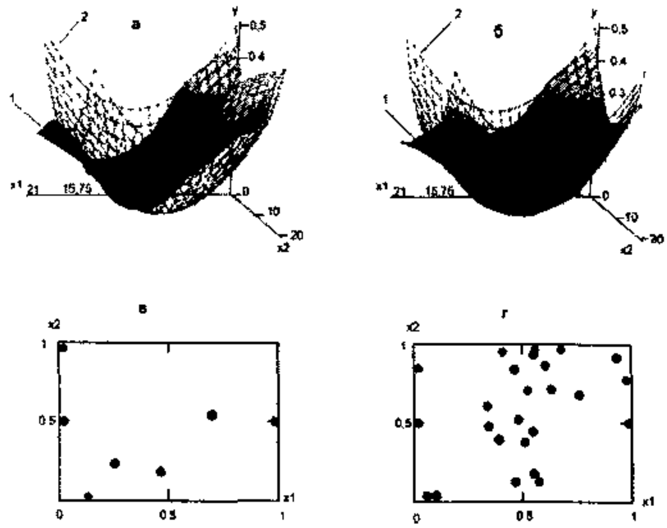

Figure 2: Approximation of function by means of superposition of wavelets in the shape of smooth delta function for no uniform distribution of interpolation nodes

2 Summary

Consequently for implementation of the principle of quality management “facts based management”, algorithm of data modeling according to the organization processes’ effectiveness has been formulated. The model of link function of the process quality indices with the influencing factors for the real conditions of its implementation constitutes the basis of the algorithm. The requirements and constraints for the model have been formulated. A universal method of approximation of passively registered quality data as an algorithm of approximation by means of superposition of wavelets in the shape of smooth delta function was suggested on the evidentiary basis.

References

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[3] Serenkov P.S., Romanchak V.M., Solomakho V.L. (2006) System of quality data collection as technical basis for functioning of efficient quality management systems. *Reports of the Acad. of Sciences of the Republic of Belarus.* Vol. **50**, No. **4**, pp. 100-104.