APPLICATION OF ROBUST DISCRIMINANT ANALYSIS FOR IMAGE STEGANALYSIS

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Abstract

In this paper we consider the application of robust discriminant analysis for image steganalysis. The efficience of statistical plug-in decision rules based on robust Huber estimator is studied.

Keywords: Steganalysis, Discriminant Analysis, Huber Estimator

1 Introduction

Statistical classification methods are widely used for image steganalysis. One approach to image steganalysis consists in application of statistical classification methods with training, notably parametric discriminant analysis and support vector machine [3].

In discriminant analysis statistical decision rules are constructed on the basis of a training sample representing observations from two classes (empty containers and modified containers). Constructed statistical decision rules are utilized for classification of newly aquired containers in one of the concerned classes.

Conditions of practical application of classical discriminant analysis usually don't hold, so the application of robust discriminant analysis is considered for image steganalysis. In this paper we study the application of robust discriminant analysis for JPEG image steganalysis.

2 Discriminant Analysis Model for Steganalysis

Consider mathematical model of steganalysis on the basis of discriminant analysis.

Let W_1 be a sample of n_1 empty containers (class Ω_1) satisfying p-dimensional Gaussian distribution $N(\mu_1, \Sigma_1)$, and W_2 be a sample of n_2 modified containers (class Ω_2) satisfying p-dimensional Gaussian distribution $N(\mu_2, \Sigma_2)$, with mean vectors $\mu_1 \in R^p$, $\mu_2 \in R^p$ and covariance matrices $\Sigma_1 \in R^{p\times p}$, $\Sigma_2 \in R^{p\times p}$, where p is the number of features. Samples W_1 and W_2 form training sample $W = W_1 \cup W_2$ of $n = n_1 + n_2$ containers.

The quadratic discriminant function for every classifying container $x \in R^p$ and for every class $i = 1,\ 2$ is defined by the following equation

$$d_i = \ln|\Sigma_i| + (x - \mu_i)^T \Sigma_i^{-1} (x - \mu_i), \quad (1)$$

where $|\Sigma_i|$ is the determinant of covariance matrix.

The decision rule consists in referring of observation $x \in R^p$ to class Ω_1, if $d_1 < d_2$, and to class Ω_2 in other case.

For linear discriminant analysis it is supposed that covariance matrices for classes Ω_1 and Ω_2 are equal: $\Sigma_1 = \Sigma_2 = \Sigma$. Therefore in linear case the discriminant function can be written in simplified form as follows

$$d_i = b_i^T x + \alpha_i, \; i = 1, 2, \tag{2}$$

where $b_i = -2\mu_i^T \Sigma^{-1}$, $\alpha_i = \mu_i^T \Sigma^{-1} \mu_i$.

If samples W_1 and W_2 don't satisfy multivariate Gaussian distribution, then robust substitution decision rules are used instead of classical statistical decision rules.

3 Classification Features

Introduce the notation to describe the feature set used in the paper [3] . Suppose the JPEG image is represented with a DCT coefficient array d_{ij}^k, $i, j = 1, \dots, 8$, $k = 1, \dots, n_B$, where n_B is the total number of DCT coefficient 8×8-blocks in the image.

Denote the global histogram of all $8 \times 8 \times n_B$ DCT coefficients as $H = (H_L, \dots, H_R)$, where $L = \min\limits_{i,j,k} d_{ij}^k$, $R = \max\limits_{i,j,k} d_{ij}^k$; and let h_r^{ij}, $r = L, \dots, R$, denotes the individual histogram of values d_{ij}^k.

Define the dual histogram for a fixed DCT coefficient value d as follows

$$g_{ij}^d = \sum_{k=1}^{n_B} \delta(d, d_{ij}^k), \tag{3}$$

where $\delta(u, v) = 1$, if $u = v$, and 0 otherwise.

Let I_r and I_c denote the vectors of block indices while scanning the image by rows and by columns respectively. As the measure capturing inter-block dependencies the variation V is used.

$$V = \frac{\sum\limits_{i,j=1}^{8} \sum\limits_{k=1}^{|I_r|-1} |d_{ij}^{I_r^{(k)}} - d_{ij}^{I_r^{(k+1)}}| + \sum\limits_{i,j=1}^{8} \sum\limits_{k=1}^{|I_c|-1} |d_{ij}^{I_c^{(k)}} - d_{ij}^{I_c^{(k+1)}}|}{|I_r| + |I_c|}. \tag{4}$$

As the measure of the discontinuities along the DCT coefficient 8×8 block boundaries the blockiness measures B_α, $\alpha = 1, 2$, are used. The blockiness measures are calculated from the decompressed JPEG image as follows

$$B_\alpha = \frac{\sum\limits_{i=1}^{\lfloor (M-1)/8 \rfloor} \sum\limits_{j=1}^{N} |x_{8i,j} - x_{8i+1,j}|^\alpha + \sum\limits_{j=1}^{\lfloor (N-1)/8 \rfloor} \sum\limits_{i=1}^{M} |x_{i,8j} - x_{i,8j+1}|^\alpha}{N\lfloor (M-1)/8 \rfloor + M\lfloor (N-1)/8 \rfloor}, \tag{5}$$

where $x_{i,j}$ are grayscale values of decompressed JPEG image, M and N are image dimensions.

The probability distribution of pairs of neighboring DCT coefficients is described by co-occurance matrix C, that defined as

$$C_{st} = \frac{\sum\limits_{k=1}^{|I_r|-1} \sum\limits_{i,j=1}^{8} \delta(s, d_{ij}^{I_r^{(k)}}) \delta(t, d_{ij}^{I_r^{(k+1)}}) + \sum\limits_{k=1}^{|I_c|-1} \sum\limits_{i,j=1}^{8} \delta(s, d_{ij}^{I_c^{(k)}}) \delta(t, d_{ij}^{I_r^{(k+1)}})}{|I_r| + |I_c|}. \tag{6}$$

The selection of informative features is performed using stepwise methods [2].

4 Robust Huber Estimator

In multivariate discriminant analysis weighted estimators of means and covariance matrices are utilized for robust parameter estimation, particularly weighted Huber estimators are used [5].

Describe the algorithm of Huber weighted parameter estimation. Suppose $X_i = (x_{i1}, \dots, x_{in_i})$ be a subsample from class Ω_i, $i = 1, 2$. And define Mahalanobis distance to the center of the subsample for every observation x_{ij} as follows

$$d_{ij} = ((x_{ij} - \bar{x}_i)^T \bar{S}_i^{-1} (x_{ij} - \bar{x}_i))^{1/2}, j = 1, \dots, n_i, \; i = 1, 2, \tag{7}$$

where $\bar{S}_i^{-1}$ is the sampling inverse covariance matrix, and $\bar{x}_i$ is the sampling mean.

Define weights w_{ij} corresponding Huber influence function and depending on distances d_{ij} as follows

$$w_{ij} = \begin{cases} 1, & d_{ij} \le d_0. \\ d_0/d_{ij}, & d_{ij} > d_0. \end{cases} \tag{8}$$

$$d_0 = \sqrt{N} + 2\sqrt{2}, j = 1, \dots, n_i, \; i = 1, 2,$$

where N is the number of variables in the model.

Build Huber weighted estimators $\hat{x}_i$, $\hat{S}_i$, $\hat{S}$ for mean vectors and covarince matrices respectively as follows

$$\hat{x}_i = \frac{\sum_{j=1}^{n_i} w_{ij} x_{ij}}{\sum_{j=1}^{n_i} w_{ij}}, \quad \hat{S}_i = \frac{\sum_{j=1}^{n_i} (w_{ij})^2 (x_{ij} - \hat{x}_i)(x_{ij} - \hat{x}_i)^T}{\sum_{j=1}^{n_i} (w_{ij})^2}, \; i = 1, 2; \tag{9}$$

It has been found experimentally that Huber estimation procedure gives good results after 5 iterations [5].

5 Experimental Results

We used a set of 1300 JPEG images as a learning sample, 650 of which contained embedded information, and 650 were empty containers. Information embedding was accomplished with steganographic algorithm F5 [6]. As a test sample we used a set of 240 JPEG images, 120 of which contained information embedded with F5, and 120 were empty containers.

Using the learning sample with stepwise methods we selected a set of 10 informative features characterizing individual and dual histograms, variation, blockiness measures and co-occurance matrix.

We carried out preliminary statistical analysis of informative features. Mulivariate normality hypothesis testing was fulfilled with Henze-Zirkler multivariate normality test [4], and its results are introduced in the table 1. According the results of Henze-Zirkler multivariate normality test the hypothesis of multivariate normality of the experimental feature set was rejected.

Covariance matrices equality hypothesis was rejected using the covariance matrices equality test [1], p-value < 0.01.

Table 1: Henze-Zirkler Multivariate Normality Test's Results

	Empty containers	Modified containers
Henze-Zirkler statistic	1.21	1.68
Henze-Zirlker p-value	<0.01	<0.01

On the experimental feature space we carried out quadratic discriminant analysis with classical and robust parameter estimation. The results of the analysis are introduced in the tables 2-3.

Table 2: Results of Quadratic Discriminant Analysis

Correct classification	Empty containers	Modified containers
Learning sample	92.6% (602/650)	94.2% (612/650)
Test sample	92.5% (111/120)	93.3% (112/120)

Table 3: Results of Robust Quadratic Discriminant Analysis

Correct classification	Empty containers	Modified containers
Learning sample	96.0% (624/650)	96.3% (626/650)
Test sample	99.2% (119/120)	97.5% (117/120)

From the results we can conclude that robust discriminant analysis is more efficient at classifying modified and empty containers than classical discriminant analysis.

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