COMPUTER ANALYSIS OF FLOW DATA FOR CONTROLLING SYSTEM REFUSALS

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Abstract

Algorithms are offered for selecting the principal elements of a system. Failures of principal elements generate an epidemic of refusals. The preventive maintenance problem of improving the reliability of functioning of a complex controlling system is considered. The statistical analysis of moments of failures of the principal elements allows to solve this reliability problem.

1 Introduction

In this work random sequences of failure instants of controlling systems functioning in extreme conditions have been studied. It is assumed that each controlling system is constructed and failures happen due to both incremental changes and unreliable functioning of the system's elements. In this case intervals between sequential failures are dependent and differently distributed. Hence one is unable to find the probability distribution for the number of controlling system's element failures $\eta(t)$ up to an arbitrary time instant $t > 0$.

A non-traditional approach of mathematical description of failures' sequence, or flow of failures, of such a complicated probabilistic structure is proposed. This approach is based on selection of so-called principal elements of controlling system. However a failure of any principal element generates a group of new failures of other elements. With examples of functioning of a large number of real controlling systems a statistical analysis of both the principal elements' failures instants and the number of failures of all types during two sequential principal elements' failures is carried out. The results of this statistical analysis let us recommend a simple method of control of a system's functioning. This method significantly reduces the variance of the number of failures of elements of all types up to moment t.

2 Mathematical statement of the problem and methods for its solution

Denote by $\{\tau'_i;\ i \geq 1\}$ a sequence of failure instants for all element types. Let now $\tau_0 = \tau'_1$ $\{\tau_i;\ i \geq 0\}$ be a strictly increasing sequence of principal elements' failures instants. Since the i-th instant $\tau_i = \tau'_{k_i}$ for $i \geq 0$, $k_i \in \{1, 2, ...\}$, $k_0 = 1$, random variable $\eta_i = k_{i+1} - k_i$ defines the number of all observed failures in a time interval

$[\tau_i, \tau_{i+1})$. Hence it is necessary to suggest algorithms to choose integer-valued random variables $\{k_i;\ i \geq 0\}$ which let us identify a probability distribution for an auxiliary vector sequence $\{(\tau_i, \eta_i);\ i \geq 0\}$. Once this problem is solved, it is natural to describe the flow of failures non-traditionally as a vector sequence $\{(\tau_i, \eta_i);\ i \geq 0\}$. In what follows we give some algorithms of constructing the sequence $\{k_i;\ i \geq 0\}$.

The fist approach. Assume that a mark of the first moment τ'_1 of failure of the element of a system coincides with mark of one of the principal elements. In this so-called synchronous case the random variables τ_i, $i \geq 0$, are defined by relations

$$\tau_i = \tau'_{k_i}, \quad k_{i+1} = \inf\{k\colon k > k_i,\ \tau'_k - \tau'_{k-1} \geq h_0\}, \quad i \geq 0,$$

where $k_0 = 1$, $h_0 = \text{const} > 0$ and τ'_1, τ'_2, ... are abscissas of points of discontinuity of the initial counting process $\{\eta(t);\ t \geq 0\}$ or moments of occurrences of events. If a set $\{k\colon k > k_i,\ \tau'_k - \tau'_{k-1} \geq h_0\} = \varnothing$ for some $i \geq 0$, we put by definition $\tau_{i+1} = +\infty$. So, this algorithm chooses the elements τ_i, $i \geq 0$ so that to each interval $[\tau_i, \tau_{i+1})$ there corresponds the i-th batch $\eta_i = k_{i+1} - k_i$. We shall remark that the formula $\eta_i = k_{i+1} - k_i$ has not a place for $\{k\colon k > k_i,\ \tau'_k - \tau'_{k-1} \geq h_0\} = \varnothing$ and finite number of random events of the initial point process τ'. However a batch η_i always defines an amount of events in $[\tau_i,\ \tau_{i+1})$. In doing so arbitrary the i-th moment τ_i coincides with some moment τ'_{k_i} of a rupture of the counting process $\{\eta(t);\ t \geq 0\}$, and intervals between any two sequential events from the i-th group are strictly less than the magnitude h_0. The events are conditionally united in batches on a principle of proximity of moments of their occurrences. Finally, the interval between a moment of occurrence of the last event from the i-th group and moment of occurrence of the first event of a batch with number $(i+1)$ is not less than magnitude h_0. We shall call this interval as an interval between two sequential batches.

The second approach. Now let the mark be the first moment τ'_1 of failure of the element of a system be not the mark of one of the principal elements. In this so-called asynchronous case the second approach to transformation of an initial flow $\{\tau'_i;\ i \geq 1\}$ is suggested. According to this approach we shall define random variables τ_i, $i \geq 0$, by equalities

$$\tau_i = \tau'_{k_i}, \quad k_0 = \inf\{k\colon k \geq 1,\ \tau'_{k+1} - \tau'_k \geq h_0\} + 1,$$
$$k_{i+1} = \inf\{k\colon k > k_i,\ \tau'_k - \tau'_{k_i} \geq h_0\}, \quad i \geq 0.$$

For the second algorithm of a choice of the point process $\{\tau_i;\ i \geq 0\}$ at first there is search for the first moment $\tau_0 = \tau'_{k_0}$ of a failure of a principal element of a system. In other words, in an initial flow $\{\tau'_i;\ i \geq 1\}$ the search of the first temporal rupture takes place. Each following moment τ_i, $i \geq 1$ is selected so that interval of time between two sequential failures of the principal elements for the first time will be not less than the given magnitude h_0. This approach to selection of the principal elements should be applied in case of an intensive flow $\{\tau'_i;\quad i \geq 1\}$ or so-called unreliable system. So, events from a flow are conditionally united in batches not only on principle of proximity of moment of their approach, but also with taking into consideration the detailed search for the principal elements.

The third approach. For each $c = 0,\ 1,\ \ldots$ we shall denote by $\tau^{(c)} = \{\tau_i^{(c)};\ i \geq 0\}$ a flow of events on the time axis $[0,\ \infty)$ defined below, which are connected to failure of the principal elements. We assume that moment $\tau_i^{(c)}$, $i \geq 0$, of this flow coincide with some points of discontinuity of the initial counting process $\{\eta(t);\ t \geq 0\}$. Then we have

$$\tau_i^{(c)} = \tau'_{k_{c,i}}, \quad k_{c,i} \in \{1, 2, ...\}.$$

Let the magnitude $\eta_i^{(c)} = k_{c,i+1} - k_{c,i}$ define the number of events in $[\tau_i^{(c)},\ \tau_{i+1}^{(c)})$ also is the i-th group of a flow $\{(\tau_i^{(c)}, \eta_i^{(c)});\ i \geq 0\}$ of events from a flow. Magnitude $\delta_i^{(c)} = \tau'_{k_{c,i+1}} - \tau'_{k_{c,i+1}-1}$ defines a time interval between sequential groups $\eta_i^{(c)}$, $\eta_{i+1}^{(c)}$ of the initial counting process $\{\eta(t);\ t \geq 0\}$ when describing as a sequence $\{\eta_i^{(c)};\ i \geq 0\}$. Then we shall build the elements $\tau_i^{(c)}$, $c \geq 0$, $i \geq 0$, of flows $\tau^{(c)}$, $c \geq 0$, of events with help of recurrent relations of the form

$$k_{0,i+1} = \inf\{k\colon k > k_{0,i},\ \tau'_k - \tau'_{k-1} \geq h_0\},$$

$$s_c = \min\{\ \inf\{\ k\colon k \geq 0,\ \eta_k^{(c)} \leq d,\ \eta_{k+1}^{(c)} = d+1,\ \delta_k^{(c)} < h_1\},$$

$$\inf\{k\colon k \geq 0,\ \eta_k^{(c)} \leq d,\ \eta_{k+1}^{(c)} \leq d,\ \delta_k^{(c)} < h_2\}\},$$

$$\tau_i^{(c+1)} = \begin{cases} \tau_i^{(c)}, & i \leq s_c, \\ \tau_{i+1}^{(c)}, & i > s_c. \end{cases}$$

In these formulae $k_{0,0} = 1$, d is a natural number, and constants h_0, h_1, h_2 satisfy to a condition $h_0 < h_1 < h_2$.

In using the third algorithm of a choice of the point process $\{\tau_i;\ i \geq 0\}$ a partition of the initial point process $\{\tau'_i\ i \geq 1\}$ happens at first by the first approach with the purpose of deriving the marked dot process $\{(\tau_i^{(0)}, \eta_i^{(0)});\ i \geq 0\}$ of a zero level. Further, sequentially starting with a zero batch $\eta_0^{(0)}$ first two adjacent groups in one of the following cases are united: a) If the previous batch contains no more than d events, the next one includes exactly $d+1$ events and simultaneously the interval between such batches is strictly less than the magnitude h_1; b) If the previous and the consequent groups contain no more than d events each and intervals between them are strictly less than the magnitude h_2. This allows to find the marked point process $\{(\tau_i^{(1)}, \eta_i^{(1)});\ i \geq 0\}$ of the first level, to which the same procedure is applied, as to the marked point process $\{(\tau_i^{(0)}, \eta_i^{(0)});\ i \geq 0\}$. In result the marked point process is obtained $\{(\tau_i^{(2)}, \eta_i^{(2)});\ i \geq 0\}$ of the second level etc.

The fourth approach. Let for $c = 0, 1, ...$ instants $\tau_i^{(c)} < \tau_{i+1}^{(c)}$, $i \geq 0$, coincide with some elements of the sequence $\{\tau'_i;\ i \geq 1\}$, i.e. the variable $\tau_i^{(c)} = \tau'_{k_{c,i}}$, $k_{c,i} \in \{1, 2, ...\}$. Then the variable $\eta_i^{(c)} = k_{c,i+1} - k_{c,i}$ determines the number of failures of all types in the interval $[\tau_i^{(c)}, \tau_{i+1}^{(c)})$. With the new description of the original flow of failures $\{\tau'_i;\ i \geq 1\}$ as a sequence $\{(\tau_i^{(c)}, \eta_i^{(c)});\ i \geq 0\}$ we call the variable $\eta_i^{(c)}$ the i-th group and the variable $\delta_i^{(c)} = \tau'_{k_{c,i+1}} - \tau'_{k_{c,i+1}-1}$ an interval between sequential groups $\eta_i^{(c)}$, $\eta_{i+1}^{(c)}$. We will construct instants $\tau_i^{(c)}$, $c \geq 0$, $i \geq 0$, with means of recurrent relations: $k_{0,i+1} = \inf\{j\colon j > k_{0,i},\ \tau'_j - \tau'_{j-1} \geq h_0\}$, $s_c = \inf\{j\colon j \geq 0,\ \eta_j^{(c)} \leq d,\ \eta_{j+1}^{(c)} \leq d,\ \delta_j^{(c)} < < h_1, \eta_j^{(c)} = \eta_{j-1}^{(c)}\}$, $\tau_i^{(c+1)} = \tau_i^{(c)}$ for $i \leq s_c$ and $\tau_i^{(c+1)} = \tau_{i+1}^{(c)}$ for $i > s_c$. In these formulae for every $c = 0, 1, \ldots$, one has $\eta_{-1}^{(c)} = 1$, $k_{0,0} = 1$, d is a natural number and the constants h_0, h_1 satisfy the conditions $h_0 < h_1$.

This algorithm for choosing sequences $\{(\tau_i^{(c)}, \eta_i^{(c)});\ i \geq 0\}$, $c = 0, 1, \ldots$, using the variable h_0 first splits the original process $\{\tau_i';\ i \geq 1\}$ into groups in order to obtain a marked point process $\{(\tau_i^{(0)}, \eta_i^{(0)});\ i \geq 0\}$ of the zero level. Then successively starting with the zero group $\eta_0^{(0)}$, the algorithm joins the first two neighboring groups $\eta_j^{(0)}$ and $\eta_{j+1}^{(0)}$, if each of them contains no more that d failures, the interval between such groups is strictly less than h_1 and finally equation $\eta_j^{(0)} = \eta_{j-1}^{(0)}$ holds. This allows to find a process $\{(\tau_i^{(1)}, \eta_i^{(1)});\ i \geq 0\}$ of the first level, to which we apply the same procedure as to the process $\{(\tau_i^{(0)}, \eta_i^{(0)});\ i \geq 0\}$. In result we obtain a marked point process $\{(\tau_i^{(2)}, \eta_i^{(2)});\ i \geq 0\}$ of the second level, and so on.

Theorem 1. *For any sample path $\omega = \{t_i';\ i \geq 1\}$ of the random sequence $\{\tau_i';\ i \geq 1\}$ and for any fixed $i \geq 0$ there exist limits $\lim_{c\to\infty} k_{c,i}(\omega)$, $\lim_{c\to\infty} \tau_i^{(c)}(\omega)$.*

This theorem allows us to define random variables $k_i = \lim_{c\to\infty} k_{c,i}$, $\tau_i = \lim_{c\to\infty}\tau_i^{(c)}$ for each $i \geq 0$. With this algorithm for choice of flow $\{(\tau_i, \eta_i);\ i \geq 0\}$ we have $\tau_i = \tau_{k_i}'$, $\eta_i = k_{i+1} - k_i$ for all $i \geq 0$ and therefore we define this way the number of all failures of the controlling system in an interval $[\tau_i, \tau_{i+1})$.

Using statistical data with observations of failure flows of real system and different hypothesis testing criteria [1] it was found that with an appropriate choice of the constants d, h_0, h_1, h_2 both the random variables $\tau_{i+1} - \tau_i$, $i \geq 0$, and the random variables η_i, $i \geq 0$, are independent and identically distributed. Moreover, probability distributions of the random variables $\tau_{i+1} - \tau_i$, η_i are defined. Using the look of random variables $\tau_{i+1} - \tau_i$, η_i and their independence, a method of routine maintainance of principal elements of the controlling system was developed which leads to increase in reliability of its functioning.

References

[1] Fedotkin A. M., Fedotkin M. A. (2004) Model for Refusals of Elements of a Controlling System. *Transactions of the first French-Russian Conference on "Longevity, Aging and Degradation Models in Reliability, Public Health, Medicine and Biology, LAD' 2004"*. St. Petersburg: St. Petersburg State Politechnical University, Vol. **2**, pp. 136–151.