ABOUT ROBUSTNESS AND POWER OF VARIANCE HOMOGENEITY TESTS

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Abstract

Classical variance homogeneity tests (Fisher's, Bartlett's, Cochran's, Hartley's and Levene's tests) and nonparametric tests for dispersion characteristics homogeneity (Ansari-Bradley's, Mood's, Siegel-Tukey's tests) have been considered. Distributions of classical tests statistics have been investigated under violation of assumption that samples belong to the normal law. The comparative analysis of power of classical variance homogeneity tests with power of nonparametric tests has been carried out. Tables of percentage points for Cochran's test have been obtained in case of distributions which are different from normal.

1 Introduction

Tests of samples homogeneity are problems of the statistical analysis often used in various applications. The question can be about checking hypotheses about homogeneity of samples distributions, population means or variances. Naturally the most complete findings can be done in the first case. However researcher can be interested in possible deviations of the sample mean values or differences in dispersion characteristics of measurements results.

One of the basic assumptions in constructing classical tests for equality of variances is normal distribution of observable random variables (measurement errors). Therefore the application of classical criteria always involves the question - how valid obtained results are in this particular situation. Under violation of assumption that analyzed variables belong to normal law, conditional distributions of tests statistics, when hypothesis under test is true, change appreciably.

This work continues researching of criteria stability for testing hypotheses about the equality of variances [1, 2]. Classical Bartlett's, Cochran's, Fisher's, Hartley's, Levene's tests are compared, nonparametric (rank) Ansari-Bradley's, Mood's, Siegel-Tukey's tests are considered [3].

The work purpose was:

- to research statistics distributions for listed tests in case of distribution laws of observable random variables which are different from normal;
- to make comparative analysis of criteria power concerning concrete competing hypotheses;
- to realize of the possibility to apply the classical tests under violation of assumptions about normality of random variables.

A hypothesis under test for equality of variances corresponding to m samples will have the form

$$H_0 : \sigma_1^2 = \sigma_2^2 = \ldots = \sigma_m^2 \quad (1)$$

and the competitive hypothesis is

$$H_1 : \sigma_{i_1}^2 \neq \sigma_{i_2}^2 \quad (2)$$

where the inequality holds at least for one pair of subscripts i_1, i_2.

Statistical simulation method and developed software were used to investigate statistics distributions, calculate percentage points and estimate tests power with respect to different competing hypotheses. The sample size of statistics under study was $N = 10^6$. Such N allowed absolute value of difference between true law of statistics distribution and simulated empirical not to exceed 10^{-3}.

Distributions of statistics were researched for different distribution laws, in particular, in case when simulated samples belong to the family with density

$$De(\theta_0) = f(x; \theta_0, \theta_1, \theta_2) = \frac{\theta_0}{2\theta_1 \Gamma(1/\theta_0)} exp\left(-\left(\frac{|x - \theta_2|}{\theta_1}\right)^{\theta_0}\right) \quad (3)$$

with different values of the form parameter θ_0. This family may be a good model for error distributions of various measuring systems. Special cases of distribution $De(\theta_0)$ include the Laplace ($\theta_0 = 1$) and normal ($\theta_0 = 2$) distribution. The family (3) allows to define various symmetric distributions that differ from normal: the smaller value of form parameter θ_0 the "heavier" tails of the distribution $De(\theta_0)$, and vice-versa the higher value the "easier" tails.

In comparative analysis of the tests power competing hypotheses with the form $H_1 : \sigma_m = d\sigma_0$ were considered. It means, that a competing hypothesis corresponds to the situation when $m - 1$ samples belong to the law with $\sigma = \sigma_0$, while one of the samples, for example, with number m has some different variance. Hypothesis under test corresponds to the situation $H_0 : \sigma_1^2 = \sigma_2^2 = \ldots = \sigma_m^2 = \sigma_0^2$.

2 Comparative analysis of power

At given probability of type I error α (to reject the null hypothesis when it is true) it is possible to judge advantages of the test by value of power $1 - \beta$, where β - probability of type II error (not to reject the null hypothesis when alternative is true). In [4] it is definitely said that Cochran's test has lower power in comparison with Bartlett's test. In [1] it was shown that Cochran's test has greater power by the example of checking hypothesis about variances homogeneity for *five* samples.

Research of power of Bartlett's, Cochran's, Hartley's, Fisher's and Levene's tests concerning such competing hypotheses $H_1 : \sigma_2 = d\sigma_1, d \neq 1$, (in case of two samples that belong to the normal law) has shown that Bartlett's, Cochran's, Hartley's and Fisher's tests have equal power in this case. Levene's test appreciably yields to them in power.

In case of the distributions which are different from normal, for example, family of distributions with density (3), Bartlett's, Cochran's, Hartley's and Fisher's tests remain equivalent in power, and Levene's test also appreciably yields to them. However in case of heavy-tailed distributions (for example, when samples belong to the Laplace distribution) Levene's test has advantage of greater power.

Bartlett's, Cochran's, Hartley's and Levene's tests can be applied when number of samples $m > 2$. In such situations power of these tests is different. If $m > 2$ and normality assumption is true, these tests can be ordered by power decrease as follows:

$$Cochran's \succ Bartlett's \succ Hartley's \succ Levene's.$$

The preference order remains in case of violation of normality assumption. The exception concerns situations when samples belong to laws with more heavy tails in comparison with the normal law. For example, in case of Laplace distribution Levene's test is more powerful than three others.

Results of nonparametric criteria power research have shown appreciable advantage of Mood's test and practical equivalence of Siegel-Tukey's and Ansari-Bradley's tests. Of course, nonparametric tests yield in power to Bartlett's, Cochran's, Hartley's and Fisher's tests. Figure 1 shows graphs of criteria power concerning competing hypotheses $H_1^1 : \sigma_2 = 1.1\sigma_1$ and $H_1^2 : \sigma_2 = 1.5\sigma_1$ depending on sample size n_i in case when $\alpha = 0.1$ and samples belong to the normal law. Advantage in power of Cochran's test is rather significant in comparison with Mood's test - most powerful of nonparametric tests. Let's remind that Bartlett's, Cochran's, Hartley's and Fisher's tests have equal power in case of two samples.

Distributions of nonparametric tests statistics do not depend on a law kind, if both samples belong to the same population. But if samples belong to different laws and hypothesis of variances equality is true, *distributions of statistics of nonparametric tests depend on a kind of these laws.*

3 Cochran's test in case of laws different from normal

Classical tests have considerable advantage in power over nonparametric. This advantage remains when analyzed samples belong to the laws appreciably different from normal. Therefore there is every reason to research statistics distributions of classical tests for checking variances homogeneity (construction of distributions models or tables of percentage points) in case of most often used laws (different from the normal law). Among considered tests Cochran's test is the most suitable for this role.

In case when observable variables belong to family of distributions (3) with parameter of the form $\theta_0 = 1, 2, 3, 4, 5$ and some values n, tables of upper percentage points (1%, 5%, 10%) for Cochran's test were obtained using statistical simulation (when number of samples $m = 2 \div 5$). Obtained results can be used in situations when distribution (3) with appropriate parameter θ_0 is a good model for observable random variables. Computed percentage points improve some results presented in [1] and expand possibilities to apply Cochran's test.

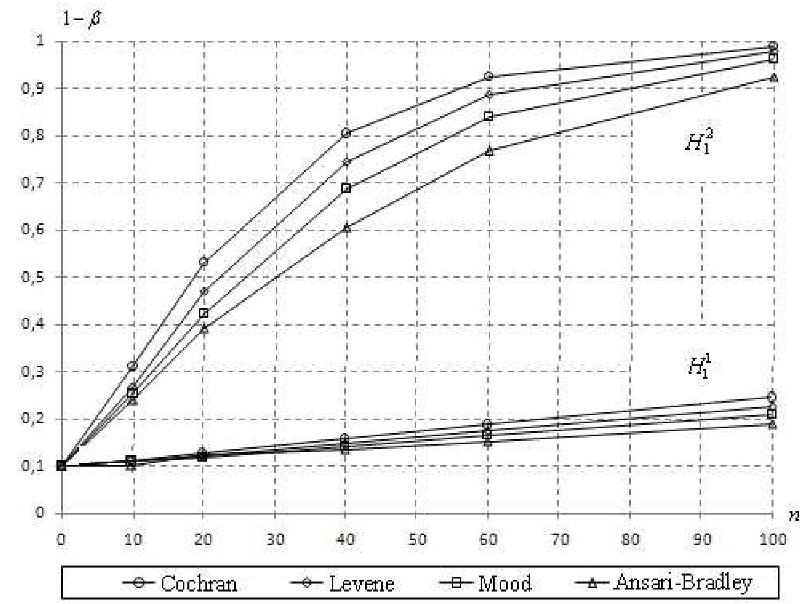

Figure 1: Power of tests concerning competing hypotheses H_1^1 and H_1^2 depending on sample size n when $\alpha = 0.1$ and samples belong to normal law

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References

[1] Lemeshko B.Yu., Mirkin E.P. (2004). Bartlett and Cochran tests in measurements with probability laws different from normal. *Measurement Techniques*. Vol. **47**, pp. 960-968.

[2] Lemeshko B.Yu., Ponomarenko V.M. (2006). Research of statistics distributions used to test hypotheses about the equality of variances in case of laws of errors different from normal (in Russian). *Scientific Journal of NSTU*. Vol. **2(23)**, pp. 21-33.

[3] Kobzar A.I. (2006). *Applied mathematical statistics. For engineers and scientists* (in Russian). FIZMATLIT Publishing House, Moscow.

[4] Bolshev L.N., Smirnov N.V. (1983). *Tables of Mathematical Statistics*. Nauka, Moscow.