Definition 1. *Equation (1) is said to be exact null-controllable on* $[0, t_1]$ *by controls vanishing after time moment* $t_2, t_2 \leq t_1$*, if for each* $x_0 \in X$ *there exists a control* $u(\cdot) \in L_2([0, t_2], U), u(t) = 0$ *a.e. on* $[t_2, +\infty)$ *such that*

$$x(t_1, x_0, u(\cdot)) = 0.$$

Necessary and sufficient conditions for the null-controllability of equation (1) are obtained by transformation of the controllability problem to some linear moment problem in Hilbert space X. The linear moment problem under consideration is formulated as follows.

Definition 2. *Given sequences* $\{c_n, n = 1, 2 \ldots, \}$ *and* $\{x_n \in X, n = 1, 2 \ldots, \}$ *find an element* $g \in X$ *such that*

$$c_n = (x_n, g), n = 1, 2 \ldots$$

Applications to the null-controllability of hyperbolic control equations with boundary control are considered.

References

1. *Hille E., Philips R.* Functional Analysis and Semi-Groups. AMS, 1957.
2. *Krein M.* Linear Differential Equations in Banach Spaces. Moscow, Nauka, 1967 (in Russian).

MINIMAX TERMINAL CONTROL PROBLEM IN TWO-LEVEL HIERARCHICAL DISCRETE-TIME DYNAMICAL SYSTEM

A.F. Shorikov

Institute of Mathematics and Mechanics Urals Branch Russian Academy of Sciences, Urals State University of Economics, 62, 8 Marta str., 620144 Ekaterinburg, Russia

shorikov@usue.ru

In this report we consider the nonlinear discrete-time dynamical system consisting of several controlled objects which has two levels of control. One level (or first level) is dominating and the other level (or second level) is subordinate which have different criteria of functioning and are united a priori by determined information and control relations. It is assumed that the motion of each object of this system is described by corresponding nonlinear recurrent vector equation (with a convex vector-values of right part of equation) and depend on both controlled parameters (controls) and non-controlled parameters (noises or simulation errors). The phase states of all objects and all a priori indeterminate parameters of this system are constrained by given convex compact sets in the corresponding Euclidean vector spaces. It is also assumed that the choice of control actions at the second level is subordinate to given conditions which depend on the choice of control actions at the first level of this control process. The quality of the control processes of all objects at both control levels in this system is estimated by corresponding convex functionals which are determine in their terminal (final) phase states and each of them meets the corresponding Lipschiz condition. Under these assumptions, we formulate the minimax program terminal control problem for processes in this two-level hierarchical discrete-time dynamical system and propose the general scheme for its solving.

At a given integer-valued time interval $\overline{0, \mathrm{T}} = \{0, 1, \cdots, \mathrm{T}\}$ $(\mathrm{T} > 0)$ we consider the controlled multi-step dynamical system and it consists of $(n+1)$ objects ($n \in \mathbf{N}$, where $\mathbf{N}$ is the set of all natural numbers). The motion of object I which is a general object and controlled by dominating player P is described by the nonlinear discrete-time recurrent vector equation

$$y(t+1) = f(t, y(t), u(t), v(t), \xi(t)), \ y(0) = y_0, \tag{1}$$

and the motion of object II_i $(i \in \overline{1,n})$ which is a subsidiary object corresponding to index i and controlled by the subordinate player E_i is described by the following nonlinear discrete-time recurrent vector equation

$$z^{(i)}(t+1) = f^{(i)}(t, z^{(i)}(t), u(t), v^{(i)}(t), \xi^{(i)}(t)), \; z^{(i)}(0) = z_0^{(i)}. \tag{2}$$

Here $t \in \overline{0, \mathrm{T}-1}$; $y \in \mathbf{R}^r$ and $z^{(i)} \in \mathbf{R}^{s_i}$ are the phase vectors of objects I and II_i respectively $(r, s_i \in \mathbf{N}$; for $k \in \mathbf{N}$, $\mathbf{R}^k$ is the k-dimensional Euclidean vector space of column vectors; $u(t) \in \mathbf{R}^p$ and $v^{(i)}(t) \in \mathbf{R}^{q_i}$ are the vectors of the control actions (controls) of players P and E_i respectively, restricted by the given constraints

$$u(t) \in \mathrm{U}_1, \; v^{(i)}(t) \in \mathrm{V}_1^{(\mathrm{i})}; \; \mathrm{U}_1 \in \mathrm{comp}(\mathbf{R}^p), \; \mathrm{V}_1^{(\mathrm{i})} \in \mathrm{comp}(\mathbf{R}^{q_i}) \; (p, \; q_i \in \mathbf{N}); \tag{3}$$

where for any $k \in \mathbf{N}$, $\mathrm{comp}(\mathbf{R^k})$ is the set of all compact subsets of the space $\mathbf{R}^k$; vector-control $v(t)$ has the form $v(t) = (v^{(1)}(t), v^{(2)}(t), \cdots, v^{(n)}(t))' \in \mathbf{R}^q$ $(q = \sum\limits_{i=1}^{n} q_i)$; $\xi(t) \in \mathbf{R}^l$ and $\xi^{(i)}(t) \in \mathbf{R}^{l_i}$ are the vectors of non-controlling parameters (noises or simulation errors) of the objects I and II_i respectively, restricted by the following given constraints

$$\xi(t) \in \Xi_1, \; \xi^{(i)}(t) \in \Xi_1^{(\mathrm{i})}; \; \Xi_1 \in \mathrm{comp}(\mathbf{R}^l), \; \Xi_1^{(\mathrm{i})} \in \mathrm{comp}(\mathbf{R}^{l_i}) \; (l, \; l_i \in \mathbf{N}); \tag{4}$$

for all $t \in \overline{0, \mathrm{T}-1}$ and $i \in \overline{1,n}$ each from the vector-functions $f : \overline{0, \mathrm{T}-1} \times \mathbf{R}^r \times \mathbf{R}^p \times \mathbf{R}^q \times \mathbf{R}^l \longrightarrow \mathbf{R}^r$ and $f^{(i)} : \overline{0, \mathrm{T}-1} \times \mathbf{R}^{s_i} \times \mathbf{R}^{q_i} \times \mathbf{R}^{l_i} \longrightarrow \mathbf{R}^{s_i}$ are continuous by collection of the variables $(y(t), u(t), v(t), \xi(t))$ and $(z^{(i)}(t), u(t), v^{(i)}(t), \xi^{(i)}(t))$ respectively.

We also assume that for all time moments $t \in \overline{0, \mathrm{T}}$ the phase vectors $y(t)$ and $z^{(i)}(t)$ of objects I and II_i $(i \in \overline{1,n})$ respectively, combined with the initial conditions in the relations (1) and (2) are restricted by the given following constraints

$$y(t) \in \mathrm{Y}_1, \; z^{(i)}(t) \in \mathrm{Z}_1^{(\mathrm{i})}; \; \mathrm{Y}_1 \in \mathrm{comp}(\mathbf{R}^r), \; \mathrm{Z}_1^{(\mathrm{i})} \in \mathrm{comp}(\mathbf{R}^{s_i}). \tag{5}$$

The control process in discrete-time dynamical system (1) – (5) are realized in the presence of the following information conditions.

In the field of interests of the player P are both possible states of object I and possible states of each objects II_i $(i \in \overline{1,n})$. And the player P also knows the formation principle of the controls $v^{(i)}(\cdot) = \{v^{(i)}(t)\}_{t \in \overline{\tau, \vartheta-1}}$ $(\forall \; t \in \overline{\tau, \vartheta-1} : \; v^{(i)}(t) \in \mathrm{V}_1^{(i)})$ of each player E_i $(i \in \overline{1,n})$ at the time interval $\overline{\tau, \vartheta}$. Then considering these circumstances we will say that such possibilities of the behaviour of player P combined with the objects I and II_i $(i \in \overline{1,n})$ are defined as the I level or the dominating level of the control process in considered system.

It is assumed that in the field of interests of each player E_i $(i \in \overline{1,n})$ are only possible states of object II_i and for any considered time interval $\overline{\tau, \vartheta}$ he also knows realization of the control $u(\cdot) = \{u(t)\}_{t \in \overline{\tau, \vartheta-1}}$ $(\forall \; t \in \overline{\tau, \vartheta-1} : \; u(t) \in \mathrm{U}_1)$ player P at this time interval, which he can use for constructing his control $v^{(i)}(t) \in \mathrm{V}_1^{(i)}$ for every time moment $t \in \overline{\tau, \vartheta-1}$.

Note, that behaviour of each player E_i $(i \in \overline{1,n})$ in achieving his aim obviously depends on the behaviour of player P. Then, granting this fact the collection n of players $E_i, \; i \in \overline{1,n}$ which will be called player E and objects $II_i, \; i \in \overline{1,n}$ controlled by them form II level or the subordinate level of control in considered system (which is subordinate to the I level or the dominating level of the control process).

The quality of the control processes of all objects at both control levels in this system is estimated by corresponding convex functionals which are determine in their terminal (final) phase states and each of them meets the corresponding Lipschiz condition.

In this report under investigation of this dynamical system we propose the mathematical formalization in the form of realization of two-level hierarchical minimax program terminal control problem and propose the general scheme for its solving.

In conclusion we note, that the concrete algorithm for realization of two-level hierarchical minimax program terminal control process in discrete-time dynamical system (1) – (5) can be described on the base of the algorithms for solving program terminal control problem which are proposed in work [4].

The results obtained in this report are based on [1] – [4] and can be used for computer simulation and for optimal digital controlling systems designing for actual technical, economic, and other multi-level control processes. Mathematical models of such systems were considered, for example, in works [1] – [3].

This work was supported by the Russian Basic Research Foundation, Project no. 07-01-00008.

References

1. *Krasovskii N.N. and Subbotin A.I.* Game-Theoretical Control Problems. Springer, Berlin, 1988.
2. *Kurzhanskii A.B.* Control and Observation under Uncertainty, Nauka, Moscow, 1977.
3. *Shorikov A.F.* Minimax Estimation and Control in Discrete-Time Dynamical Systems. Urals State University Publisher, Ekaterinburg, 1997.
4. *Shorikov A.F.* Algorithm for Solving a Problem of ϵ-Optimal Program Control for Descrete-Time Dynamical System // Control Theory and Theory of Generalized Solutions of Hamilton–Jacobi Equations. Abstracts of International Seminar. Urals State University Publisher, Ekaterinburg, 2005. P. 176–178.

STUDY OF REVERSE MAGNUS EFFECT IN THE CASE OF SPINNING ROUGH DISK MOVING IN A RARIFIED MEDIUM

T. Tchemisova[1], A. Plakhov[2]

[1] University of Aveiro, Aveiro 3810, Portugal
tatiana@ua.pt

[2] University of Wales - Aberystwyth, Aberystwyth SY23 3BZ, Ceredigion, UK;
on leave from University of Aveiro, Portugal
axp@aber.ac.uk

We are concerned with a spinning solid body moving in a homogeneous medium. The medium is extremely rarefied, so the free path length of the medium particles is much larger than the body's size. In such a case, the interaction of the body with the medium can be described in terms of *free molecular flow*, where a flow of point particles falls on the body's surface; each particle interacts with the body and does not with other particles. We suppose that the medium particles stay at rest, that is, the absolute temperature of the medium equals zero. In a frame of reference moving forward together with the body, we have a parallel flow of particles falling on the body.

We neglect the angular momentum of particles; each particle is identified with a mass point that approaches the body, makes several (maybe none) reflections from its surface, and then goes away. All reflections are supposed to be *absolutely elastic*.

In this paper, we restrict ourselves to the two-dimensional case. Consider a body contained in a circle of radius r and containing the concentric circle of radius $r - \varepsilon$ with $\varepsilon \ll r$. One can imagine a circle of radius r slightly damaged near the boundary, resulting in a *rough circle*. The shape of the body cannot change in time: it can only be translated or rotated. Denote by $\varphi(t)$ the rotation angle at the moment t, by $\omega(t)$ the angular velocity of the body, $\omega(t) = d\varphi/dt$, and let $\vec{v}(t)$ be the velocity of the body's center of mass. Let us agree to count off the rotation angle and the angular velocity clockwise.

Here we consider two problems as follows: (i) determine the force of the medium resistance acting on the body, find the moment of this force with respect to the body's center of mass, and investigate their dependence on the "shape of the roughness", and (ii) analyze the motion of the body in the medium, that is, study the functions $\omega(t)$ and $\vec{v}(t)$. Problem (i) is primary with