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ON THE CONVEXITY OF REACHABLE SETS FOR CERTAIN CONTROL PROBLEMS – AN ERRATUM

W.H. Schmidt[1], W. Korobow[2], G. Sklyar[3]

[1] University of Greifswald, Dep. Math. and Computer Science, Jahnstrasse 15a, 17487, Greifswald, Germany
wschmidt@uni-greifswald.de

[2] Kharkov National University, Dep. Diff. Equations and Control, Svobody sqr. 4, 61077, Kharkov, Ukraine

[3] Szczecin University, Wielkopolska Str. 15, 70451, Szczecin, Poland

We learnt in the textbook "Optimization of linear systems" by R. F. Gabasov and F. M. Kirillova (Minsk 1973) that the reachable set of a control system described by

$$\dot{x}(t) = A(t)x(t) + b(t, u(t)), \quad u(t) \in U,$$

is convex, even if the control set U is not convex. The convexity of the reachable set of a more general problem is insisted in the paper "On Convexity of Reachable Sets for Nonlinear Control Systems" presented by V. Azhmyakov, D. Flockerzi and J. Raisch, 2007. The controls are assumed to be Lipschitz–functions with Lipschitz–constants $l_u \leq l$ and the right–hand side f of the process equation is assumed to fulfil a Lipschitz–condition. However, the main theorem of this paper dealing with the convexity of the reachable set is false. We will illustrate this by means of two counterexamples.

ON EXACT NULL-CONTROLLABILITY OF DISTRIBUTED SYSTEMS

B. Shklyar

Holon Institute of Technology, 52 Golomb Str., P.O.B. 305, 58102 Holon, Israel
shk_b@hit.ac.il

Let X, U be Hilbert spaces, and let A be infinitesimal generator of strongly continuous C_0-semigroups $S(t)$ in X [1],[2]. Consider the abstract evolution control equation [2]

$$\dot{x}(t) = Ax(t) + Bu(t), x(0) = x_0, \ \ 0 \leq t < +\infty, \tag{1}$$

where $x(t), \ x_0 \in X, u(t), u_0 \in U, \ B : U \to X$ is a linear possibly unbounded operator, $W = D(A) \subset X \subset V$ are also Hilbert spaces with continuous dense injections and $A : W \to V$ and $B : U \to V$ are bounded operators. The space W is a Hilbert space with respect of graph norm.

Let $x(t, x_0, u(\cdot))$ be a mild solution of equation (1) with initial condition $x(0) = x_0$.

The assumptions on A are listed below.

1. The operators A has purely point spectrums σ with no finite limit points. All eigenvalues of A have finite multiplicities.

2. There exists $T \geq 0$ such that all mild solutions of the equation $\dot{x}(t) = Ax(t)$ are expanded in a series of generalized eigenvectors of the operator A converging uniformly for any $t \in [T_1, T_2], T < T_1 < T_2$.

Definition 1. *Equation (1) is said to be exact null-controllable on* $[0, t_1]$ *by controls vanishing after time moment* $t_2, t_2 \leq t_1$, *if for each* $x_0 \in X$ *there exists a control* $u(\cdot) \in L_2([0, t_2], U), u(t) = 0$ *a.e. on* $[t_2, +\infty)$ *such that*

$$x(t_1, x_0, u(\cdot)) = 0.$$

Necessary and sufficient conditions for the null-controllability of equation (1) are obtained by transformation of the controllability problem to some linear moment problem in Hilbert space X. The linear moment problem under consideration is formulated as follows.

Definition 2. *Given sequences* $\{c_n, n = 1, 2 \ldots,\}$ *and* $\{x_n \in X, n = 1, 2 \ldots,\}$ *find an element* $g \in X$ *such that*

$$c_n = (x_n, g), n = 1, 2 \ldots$$

Applications to the null-controllability of hyperbolic control equations with boundary control are considered.

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MINIMAX TERMINAL CONTROL PROBLEM IN TWO-LEVEL HIERARCHICAL DISCRETE-TIME DYNAMICAL SYSTEM

A.F. Shorikov

Institute of Mathematics and Mechanics Urals Branch Russian Academy of Sciences, Urals State University of Economics, 62, 8 Marta str., 620144 Ekaterinburg, Russia

shorikov@usue.ru

In this report we consider the nonlinear discrete-time dynamical system consisting of several controlled objects which has two levels of control. One level (or first level) is dominating and the other level (or second level) is subordinate which have different criteria of functioning and are united a priori by determined information and control relations. It is assumed that the motion of each object of this system is described by corresponding nonlinear recurrent vector equation (with a convex vector-values of right part of equation) and depend on both controlled parameters (controls) and non-controlled parameters (noises or simulation errors). The phase states of all objects and all a priori indeterminate parameters of this system are constrained by given convex compact sets in the corresponding Euclidean vector spaces. It is also assumed that the choice of control actions at the second level is subordinate to given conditions which depend on the choice of control actions at the first level of this control process. The quality of the control processes of all objects at both control levels in this system is estimated by corresponding convex functionals which are determine in their terminal (final) phase states and each of them meets the corresponding Lipschiz condition. Under these assumptions, we formulate the minimax program terminal control problem for processes in this two-level hierarchical discrete-time dynamical system and propose the general scheme for its solving.

At a given integer-valued time interval $\overline{0, \mathrm{T}} = \{0, 1, \cdots, \mathrm{T}\}$ $(\mathrm{T} > 0)$ we consider the controlled multi-step dynamical system and it consists of $(n+1)$ objects ($n \in \mathbf{N}$, where $\mathbf{N}$ is the set of all natural numbers). The motion of object I which is a general object and controlled by dominating player P is described by the nonlinear discrete-time recurrent vector equation

$$y(t+1) = f(t, y(t), u(t), v(t), \xi(t)), \; y(0) = y_0, \tag{1}$$