3. Results. On the basis of the aforecited discourses using the methods of convex and non-smooth analysis we determined following facts.

Теорема 1. *Let $x_0 \in Q$ is the solution of the problem 4. If the maps $F : X \to (W, C), G : X \to (Y, K)$ are differentiable , and the map $H : X \to Z$ is continuously differentiable by Freshet in the point x_0, then there exists the non-zero element $(\eta^*, \mu^*, \nu^*) \in C^* \times K^* \times Z^*$, satisfying the conditions:*

$$\eta^* \circ F'(x_0) + \mu^* \circ G'(x_0) + \nu^* \circ H'(x_0) = 0, \langle \mu^*, G(x_0) \rangle = 0. \qquad (5)$$

Теорема 2. *Let $x_0 \in Q$ is the solution of the problem 4. X is a Banach space, and Y, Z, W are Hylbert spaces. If the maps $F : X \to (W, C), G : X \to (Y, K), H : X \to Z$ are locally Lipsehitz on X, then there exists the non-zero element $(\eta^*, \mu^*, \mu^*) \in C^* \times K^* \times Z^*$, satisfying the conditions:*

$$0 \in (\eta^* \circ F + \mu^* \circ G + \nu^* \circ H)(x_0), (\mu^* \circ G)(x_0) = 0. \qquad (6)$$

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ON THE OBSERVABILITY OF SMALL SOLUTIONS OF LINEAR DIFFERENTIAL-ALGEBRAIC SYSTEMS WITH DELAYS

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Introduction. The behaviour of a number of real physics process consists of a combination of dynamic (differential) and algebraic (functional) dependencies. These processes are described by differential-algebraic systems. In that sense these systems are hybrid systems. It should be noted that the term "hybrid systems"has been widely used in the literature in various senses [1].

The paper deals with the weak observability of small solutions of DAD systems; it is an extension of the work [2]. The small solution is a solution that goes to zero faster than any exponential function. Existence of such solutions for linear retarded systems was proved by Henry [3] and later by Kappel [4] for linear neutral type systems. Lunel [5] gave explicit characterization of the smallest possible time for which small solutions vanish. Observability of small solutions for the retarded time delay system case was first studied by Manitius [6] and for general neutral system by Salamon [7].

1. Preliminaries. Let us consider DAD system in the form

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{12}x_2(t), \ \ t > 0, \tag{1a}$$

$$x_2(t) = A_{21}x_1(t) + A_{22}x_2(t-h), \ \ t \geq 0, \tag{1b}$$

$$z(t) = B_1x_1(t) + B_2x_2(t). \tag{1c}$$

Here $x_1(t) \in \mathbb{R}^{n_1}$, $x_2(t) \in \mathbb{R}^{n_2}$, $z(t) \in \mathbb{R}^m$, $t \geq 0; A_{11} \in \mathbb{R}^{n_1\times n_1}$, $A_{12} \in \mathbb{R}^{n_1\times n_2}$, $A_{21} \in \mathbb{R}^{n_2\times n_1}$, $A_{22} \in \mathbb{R}^{n_2\times n_2}$, $B_1 \in \mathbb{R}^{m\times n_1}$, $B_2 \in \mathbb{R}^{m\times n_2}$, are constant matrices, h is a constant delay, $h > 0$. We regard an absolute continuous n_1-vector function $x_1(\cdot)$, and a piecewise continuous n_2-vector function $x_2(\cdot)$ as the solutions of systems (1) if they satisfy the equations (1a)-(1b). System (1) should be completed with initial conditions in the form:

$$x_1(0+) = x_1(0) = x_{10}, \ x_2(\tau) = \psi(\tau), \ \tau \in [-h, 0),$$

where $\psi \in PC([-h, 0], \mathbb{R}^m)$ and $PC([-h, 0], \mathbb{R}^m)$ is the set of piecewise continuous m-vector functions in $[-h, 0]$.

Definition 1. *We say that a solution $x_1(\cdot)$, $x_2(\cdot)$ is small, if there exists $T > 0$ such that $x_1(t) = 0$, $x_2(t) = 0$ for $t \geq T$. A small solution is trivial, if $x_1(t) = 0$ and $x_2(t) = 0$ for $t \geq 0$.*

Similarly we define the notion of small solutions with respect to x_1 and x_2.

2. Relative observability of small solutions.

Definition 2. *Nontrivial small solutions of system (1) with respect to x_2 are weakly observable if every nontrivial small solution with respect to x_2 has nonzero output for $t \geq 0$ and x_1 is a zero solution, i.e.*

$$\exists T > 0 \quad \left.\begin{array}{c} x_1(t) = 0\,\forall t \geq 0 \\ x_2(t) = 0\,\forall t \geq T \\ z(t) = 0\,\forall t \geq 0 \end{array}\right\} \Rightarrow x_2(t) = 0, \ \forall t \geq 0.$$

Theorem 1. *Nontrivial small solutions of system (1) with respect to x_2 are observable if and only if the following condition holds:*

$$\operatorname{rank}\begin{bmatrix} B_2A_{22} \\ B_2(A_{22})^2 \\ \vdots \\ B_2(A_{22})^{n_2} \end{bmatrix} = \operatorname{rank}\begin{bmatrix} B_2A_{22} \\ B_2(A_{22})^2 \\ \vdots \\ B_2(A_{22})^{n_2} \\ A_{22} \end{bmatrix}.$$

Definition 3. *We say that $x_1(t)$, $t > 0$, $x_2(t)$, $t > 0$ is a strong solution of system (1) if equations (1) are satisfied for all t, $t > 0$ (in (1) we consider the right-hand derivative at $t = 0$).*

Theorem 2. *Nontrivial strong small solutions of system (1) with respect to x_2 are observable if and only if the following condition holds:*

$$\operatorname{rank}\begin{bmatrix} A_{12}A_{22} \\ \vdots \\ A_{12}A_{22}^{n_2} \\ B_2A_{22} \\ \vdots \\ B_2A_{22}^{n_2} \end{bmatrix} = \operatorname{rank}\begin{bmatrix} A_{12}A_{22} \\ \vdots \\ A_{12}A_{22}^{n_2} \\ B_2A_{22} \\ \vdots \\ B_2A_{22}^{n_2} \\ A_{22} \end{bmatrix}.$$

Corollary 1. *If nontrivial small solutions of system (1) with respect to x_2 are observable then nontrivial strong small solutions of system (1) with respect to x_2 of system (1) are also observable.*

Definition 4. *Nontrivial small solutions of system (1) with respect to x_1 are weakly observable if every nontrivial small solution with respect to x_1 has nonzero output for $t \geq 0$ and for x_2 being zero solution, i.e.*

$$\exists T > 0 \quad \left.\begin{array}{c} x_2(t) = 0\,\forall t \geq 0 \\ x_1(t) = 0\,\forall t \geq T \\ z(t) = 0\,\forall t \geq 0 \end{array}\right\} \Rightarrow x_1(t) = 0,\ \forall t \geq 0.$$

Theorem 3. *Nontrivial small solutions of system (1) with respect to x_1 are always observable.*

Conclusion. In this paper we have investigated the problem of relative weak observability of nontrivial small solutions of the hybrid differential-difference systems. Weak observability of nontrivial small solution with respect to x_2 and x_1 are considered. Strong small solutions are defined and weak observability of nontrivial strong small solutions with respect to x_2 is established. Other kinds of observability of small solution of system (1) and relations between these kinds of observability are also discussed.

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PORT-CONTROLLED HAMILTONIAN SYSTEMS WITH NONHOLONOMIC CONSTRAINS IMPOSED BY CONTROL DESIGN OBJECTIVES

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This paper investigates the Port-Controlled Hamiltonian (PCH) model of nonholonomic dynamical systems [4]. Nonholonomic constraints on motion can be expressed in terms of nonintegrable linear velocity relationships $B(q)\dot{q} = 0$. Recently [2], this class of nonholonomic constraints has been broadened as to encompass affine velocity relationships $A(q) + B(q)\dot{q} = 0$. PCH model incorporates directly nonholonomic constraints and broadening the class of nonholonomic constraints allows us to propose in a PCH formulation a control algorithm for dynamical systems where nonholonomic constraints on velocities are imposed by control objectives [1] and not by the structure of the system itself [3]. As an application, the energy based robust control is studied of finite dimensional underactuated mechanical systems.