The weak stationarity part of the last statement is related to fundamental concepts of variational analysis. It recaptures in the Asplund space setting the *Extended Extremal Principle* and strengthens the *Extremal Principle*.

Extremal Principle ([6, 7]) *If the collection of sets* $\Omega_1, \Omega_2, \ldots, \Omega_n$ *is locally extremal at* x° *then* $\eta[\Omega_1, \ldots, \Omega_n](x^\circ) = 0$.

Extended Extremal Principle ([2]) *The collection of sets* $\Omega_1, \Omega_2, \ldots, \Omega_n$ *is weakly stationary at* x° *if and only if* $\eta[\Omega_1, \ldots, \Omega_n](x^\circ) = 0$.

Taking into account the extremal characterizations of Asplund spaces in [7] one can formulate the following theorem.

Theorem 3 ([3]) *The following assertions are equivalent:*

1. *X is an Asplund space;*
2. *The Extremal principle is valid in X;*
3. *The Extended extremal principle is valid in X.*

References

1. *Kruger A.Y.* On extremality of set systems // Dokl. Nats. Akad. Nauk Belarusi. 1998. V. 42. No. 1. P. 24–28 (in Russian).
2. *Kruger A.Y.* On Fréchet subdifferentials // J. Math. Sci. 2003. V. 116. No. 3. P. 3325–3358.
3. *Kruger A.Y.* Stationarity and regularity of set systems // Pac. J. Optim. 2005. V. 1. No. 1. P. 101–126.
4. *Kruger A.Y.* Weak stationarity: eliminating the gap between necessary and sufficient conditions // Optimization. 2004. V. 53. No. 2. P. 147–164.
5. *Kruger A.Y.* About regularity of collections of sets // Set-Valued Anal. 2006. V. 14. No. 2. P. 187–206.
6. *Kruger A.Y., Mordukhovich B.S.* Extremal points and the Euler equation in nonsmooth optimization // Dokl. Akad. Nauk BSSR. 1980. V. 24. No. 8. P. 684–687 (in Russian).
7. *Mordukhovich B.S., Shao Y.* Extremal characterizations of Asplund spaces // Proc. Amer. Math. Soc. 1996. V. 124. P. 197–205.

CONDITIONS OF CONIC EXTREMALITY

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Introduction. The paper is dedicated to the problems of non-scalar optimization, i.e. to the problems of vector optimization, in which a criteria space is not necessarily finite-dimensional. There are two motives of the consideration of such problems: the classical concepts of the solution of the vector optimization problem don't allow the direct generalization in case of infinite-dimensional spaces [1]; there is a connection between the theory of mathematical games and the non-scalar optimization.

1. Basic concepts and facts. The reflexive and transitive relation $\succcurlyeq$ is a linear partial order in the linear space $\mathfrak{W}$, if $a \succcurlyeq b \Rightarrow a + c \succcurlyeq b + c, ta \succcurlyeq tb. \forall a, b, c \in \mathfrak{W}, t > 0$. For the linear partial order $\succcurlyeq$ the set $C = \{a \in \mathfrak{W} | a \succcurlyeq 0\}$ is a convex cone with the vertex $0 \in \mathfrak{W}$. Conversely, the convex cone $C \subset \mathfrak{W}$ with the zero vertex in the linear space $\mathfrak{W}$ induces the linear order $\succcurlyeq$ by the condition $a \succcurlyeq b \Leftrightarrow a - b \in \mathfrak{W}$. The partially ordered linear space $\mathfrak{W}$ with the convex cone $C \subset \mathfrak{W}, 0 \in C$. Below, without a special stipulation, the cone $C \subset \mathfrak{W}$ is considered sharp, i.e. $C \cap (-C) = \{0\}$.

Определение 1. *Let $\mathfrak{W}$ - is a linear topological space and $\Omega \subset (\mathfrak{W}, C)$ is a nonempty subset. The element $w_0 \in \Omega$ is called C - minimal point of the set Ω, if $\Omega \cap (w_0 - C) = \{w_0\}$. If $intC \neq \emptyset$ and there exists the ratio $\Omega \cap (w_0 - intC) = \emptyset$, then w_0 is a weakly C - minimal point of the set Ω.*

The properties of just defined conic extremum points of the nonempty subset of the linear topological space and the conditions of existence are determined in [2]. The presented paper refers to the search of conically extremum points in the weak sense. Using the concept of a tangent vector to the nonempty subset V of the Banach space $\mathfrak{X}$ in the point $x_0 \in V$, the tangent cone $T_V(x_0)$ - a closed cone with a zero vertex in $\mathfrak{X}$, we can write as

$$T_V(x_0) = \{v \in \mathfrak{X}\backslash\{0\} | \exists \{x_n\} \subset V\backslash\{x_0\}, x_n \to x_0, n \to \infty, v = \lim_{n\to\infty}(x_n - x_0)\backslash||x_n - x_0||\} \cup \{0\}.$$

If $S : U \to \mathfrak{W}$ - a continuously differentiable map by Freshet of the neighbourhood U of the point x_0 of the Banach space $\mathfrak{X}$ into the Banach space $\mathfrak{W}$ satisfies the condition of regularity $S'(x_0)\mathfrak{X} = \mathfrak{W}$, and $V = \{x \in U | S(x) = S(x_0)\}$, according to the Lusternic theorem [3], the tangent cone $T_V(x_0)$ coincides with the kernel of the operator $S'(x_0) \in L(\mathfrak{X}, \mathfrak{W})$. In other words we have the equality

$$T_V(x_0) = \{v \in \mathfrak{X} | S'(x_0)v = 0\}. \tag{1}$$

Определение 2. *The map $S : X \to \mathfrak{W}$ of the normed space $\mathfrak{X}$ in the normmed space $\mathfrak{W}$ is called locally Lipschitz on $\mathfrak{X}$, if for any $x \in \mathfrak{W}$ there exist the neighbourhood U and the number $\sigma = \sigma(U) > 0$, for which*

$$||S(u) - S(h)||_{\mathfrak{W}} \leq \sigma ||u - h||_{\mathfrak{X}}, \forall u, h \in U.$$

Определение 3. *Let B is an open unit sphere in the Banach space $\mathfrak{X}$, $w : \mathfrak{X} \to R$ is a real function $(x, v) \in \mathfrak{X} \times \mathfrak{X}$. The number $w^0(x, v)$, defined by the equality*

$$w^0(x, v) = \inf_{\delta>0} \sup_{y \in x+\delta B} \frac{w(y + tv) - w(y)}{t}, \tag{2}$$

is called the Clarke derivative of function w in the point x in the direction v.

We can verify that if $w : \mathfrak{X} \to R$ is locally Lipsehitz on $\mathfrak{X}$, then the map $v \to w^0(x, v)$ is finite, positively homogeneous and subadditive on $\mathfrak{X}$, wherefrom and from Hahn-Banach theorem it follows that the generalized Clarke gradient

$$\partial_C(x) = \{x^* \in \mathfrak{X}^* | \langle x^*, v\rangle \leq w^0(x, v), \forall v \in \mathfrak{X}\}$$

is a nonempty subset of the conjugate space X^* to the space X [4]. For the metric spaces the variational Ekeland principle [5], is valid, from which for the Banach spaces it follows: if the function $\varphi : \mathfrak{X} \to R$ is positive and lower semicontinuos on $\mathfrak{X}$, and some elements $x_0 \in \mathfrak{X}, t > 0$ satisfy the inequality $\varphi(x_0) \leq \inf_{x\in\mathfrak{X}} \varphi(x) + t$, then there exists $x_t \in \mathfrak{X}$ such that:

$$\varphi(x_t) \leq \varphi(x_0), ||\varphi(x_t) - \varphi(x_0)|| \leq \sqrt{t}, \varphi(x_t) < \varphi(x) + \sqrt{t}||x - x_t||, \forall x \neq x_t. \tag{3}$$

2. Problem setting. Let X, Y, Z, W are Banach spaces, Y, W are partially ordered by convex closed sharp bodily cones $K \subset Y, C \subset W$ and the maps $F : X \to (W, C), G : X \to (Y, K), G : X \to Z$ are given. The problem of non-scalar optimization is as follows:

$$\min_{x\in Q} F(x), Q = \{x \in X | G(x) \in -K, H(x) = 0\}. \tag{4}$$

By the solution of the problem 4 we understand any weakly C - minimal vector of the set Q.

3. Results. On the basis of the aforecited discourses using the methods of convex and non-smooth analysis we determined following facts.

Теорема 1. *Let $x_0 \in Q$ is the solution of the problem 4. If the maps $F : X \to (W, C), G : X \to (Y, K)$ are differentiable , and the map $H : X \to Z$ is continuously differentiable by Freshet in the point x_0, then there exists the non-zero element $(\eta^*, \mu^*, \nu^*) \in C^* \times K^* \times Z^*$, satisfying the conditions:*

$$\eta^* \circ F'(x_0) + \mu^* \circ G'(x_0) + \nu^* \circ H'(x_0) = 0, \langle \mu^*, G(x_0) \rangle = 0. \tag{5}$$

Теорема 2. *Let $x_0 \in Q$ is the solution of the problem 4. X is a Banach space, and Y, Z, W are Hylbert spaces. If the maps $F : X \to (W, C), G : X \to (Y, K), H : X \to Z$ are locally Lipsehitz on X, then there exists the non-zero element $(\eta^*, \mu^*, \mu^*) \in C^* \times K^* \times Z^*$, satisfying the conditions:*

$$0 \in (\eta^* \circ F + \mu^* \circ G + \nu^* \circ H)(x_0), (\mu^* \circ G)(x_0) = 0. \tag{6}$$

References

1. *Salukvadze M.E.* Vector - Valued Optimization Problems in Control Theory // N.Y. etc.: Academic Press, 1979.
2. *Maisuradze V.G.* On some questions of non-scalar optimization //Proceedings of the A.I.Eliashvili Institute of Control Systems of the Georgian Academy of Sciences, N 10, Tbilisi, 2006. (Rus.)
3. *Ioffe A.D, Tikhomirov V.M.* Theory of extremum problems // Nauka, Moscow, 19784.(Rus.)
4. *Clarke F.H.* Optimization and nonsmooth analisis // Wiley-Intersciense, New York, 1983.
5. *Aubin J.-P.,Ekeland I.* Applied nonlinear analysis // Mir, Moscow, 1988. (Rus.)

ON THE OBSERVABILITY OF SMALL SOLUTIONS OF LINEAR DIFFERENTIAL-ALGEBRAIC SYSTEMS WITH DELAYS

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Introduction. The behaviour of a number of real physics process consists of a combination of dynamic (differential) and algebraic (functional) dependencies. These processes are described by differential-algebraic systems. In that sense these systems are hybrid systems. It should be noted that the term "hybrid systems"has been widely used in the literature in various senses [1].

The paper deals with the weak observability of small solutions of DAD systems; it is an extension of the work [2]. The small solution is a solution that goes to zero faster than any exponential function. Existence of such solutions for linear retarded systems was proved by Henry [3] and later by Kappel [4] for linear neutral type systems. Lunel [5] gave explicit characterization of the smallest possible time for which small solutions vanish. Observability of small solutions for the retarded time delay system case was first studied by Manitius [6] and for general neutral system by Salamon [7].