The paper deals with the peculiarities of dynamic model of the autonomous underwater robot [4] and implementation of programmed motion taking into account contradictory requirements of stability and accuracy depth stabilization or equidistant curve with regard to relief. It also gives examples of practical realization of the offered solutions in the structure and algorithms of motion control of certain autonomous underwater vehicles-robots.

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GLOBAL POSITIONAL SYNTHESIS AND FINITE-TIME STABILIZATION OF MIMO GENERALIZED TRIANGULAR SYSTEMS BY MEANS OF CONTROLLABILITY FUNCTION METHOD

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We solve the problem of global stabilization in finite time by means of a continuous feedback law for a general class of tridiagonal multi-input and multi-output systems with singular input-output links. We combine the controllability function method (which works locally around the equilibrium) with a modification of global construction developed in previous works for the singular case.

ABOUT EXTREMALITY, STATIONARITY, AND REGULARITY

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Introduction. Extremality, stationarity, and regularity properties of collections of sets in a normed linear space are characterized in terms of certain primal and dual constants.

1. Primal Constants. Consider a collection of sets $\Omega_1, \Omega_2, \ldots, \Omega_n$ $(n > 1)$ in a normed space X with $x^\circ \in \cap_{i=1}^n \Omega_i$. The following nonnegative (not necessarily finite) constants can be used for characterizing the mutual arrangement of the sets near x° [1, 2, 4, 3, 5]:

$$\theta_\rho[\Omega_1, \ldots, \Omega_n](x^\circ) = \sup\{r \geq 0 : \left(\bigcap_{i=1}^n (\Omega_i - a_i)\right) \bigcap (x^\circ + \rho B) \neq \emptyset,\ \forall a_i \in rB\},$$

$$\theta[\Omega_1, \ldots, \Omega_n](x^\circ) = \liminf_{\rho \to +0} \frac{\theta_\rho[\Omega_1, \ldots, \Omega_n](x^\circ)}{\rho},$$

$$\hat{\theta}[\Omega_1, \ldots, \Omega_n](x^\circ) = \liminf_{\substack{\omega_i \xrightarrow{\Omega_i} x^\circ \\ \rho \to +0}} \frac{\theta_\rho[\Omega_1 - \omega_1, \ldots, \Omega_n - \omega_n](0)}{\rho}.$$

Definition 1. *The collection of sets $\Omega_1,\ \Omega_2,\ \ldots,\ \Omega_n$ is*

1. *extremal at x° if $\theta_\rho[\Omega_1,\ldots,\Omega_n](x^\circ)=0$ for all $\rho>0$.*
2. *locally extremal at x° if $\theta_\rho[\Omega_1,\ldots,\Omega_n](x^\circ)=0$ for some $\rho>0$.*
3. *stationary at x° if $\theta[\Omega_1,\ldots,\Omega_n](x^\circ)=0$.*
4. *weakly stationary at x° if $\hat{\theta}[\Omega_1,\ldots,\Omega_n](x^\circ)=0$.*
5. *regular at x° if $\theta[\Omega_1,\ldots,\Omega_n](x^\circ)>0$.*
6. *strongly regular at x° if $\hat{\theta}[\Omega_1,\ldots,\Omega_n](x^\circ)>0$.*

Items (i), (ii) of the above definition recapture the original definitions from [6]. The following implications are straightforward: (i) $\Rightarrow$ (ii) $\Rightarrow$ (iii) $\Rightarrow$ (iv), (vi) $\Rightarrow$ (v).

2. Metric Constant. Strong regularity (or weak stationarity) of the collection of sets can be also characterized using the next "metric" constant [3, 5]:

$$\hat{\vartheta}[\Omega_1,\ldots,\Omega_n](x^\circ)=\limsup_{\substack{x\to x^\circ\\ x_i\to 0}}\left[d(x,\bigcap_{i=1}^{n}(\Omega_i-x_i))\Big/\max_{1\le i\le n}d(x+x_i,\Omega_i)\right]_\circ.$$

Here $d(\cdot,\cdot)$ is the point-to-set distance in X and $[\cdot/\cdot]_\circ$ denotes the "extended"division operation. It differs from the usual one in the following additional rule: $[0/0]_\circ=0$.

Theorem 1 ([3]) $\hat{\vartheta}[\Omega_1,\ldots,\Omega_n](x^\circ)=1/\hat{\theta}[\Omega_1,\ldots,\Omega_n](x^\circ)$.

3. Dual Constants. In this section the sets $\Omega_1,\Omega_2,\ldots,\Omega_n$ are assumed closed.

Recall that the *Fréchet normal cone* to a set Ω at a point $x^\circ\in\Omega$ is defined as

$$N(x^\circ|\Omega)=\left\{x^*\in X^*:\ \limsup_{x\xrightarrow{\Omega}x^\circ}\frac{\langle x^*,x-x^\circ\rangle}{\|x-x^\circ\|}\le 0\right\}.$$

Here X^* is the space (topologically) dual to X, $\langle\cdot,\cdot\rangle$ is the bilinear form defining duality between X and X^* and $x\xrightarrow{\Omega}x^\circ$ means that $x\to x^\circ$ while $x\in\Omega$.

Using Fréchet normal cones one can define a dual constant:

$$\eta[\Omega_1,\ldots,\Omega_n](x^\circ)=\lim_{\delta\to+0}\inf\left\{\left[\left\|\sum_{i=1}^{n}x_i^*\right\|\Big/\sum_{i=1}^{n}\|x_i^*\|\right]_\infty:\right.$$
$$\left.x_i^*\in N(x_i|\Omega_i),\ x_i\in\Omega_i\cap(x^\circ+\delta B),\ i=1,\ldots,n\right\}.$$

Here we use another "extended"division operation $[\cdot,\cdot]_\infty$ with the rule $[0/0]_\infty=\infty$.

Theorem 2 ([4]) $\hat{\theta}[\Omega_1,\ldots,\Omega_n](x^\circ)\le\eta[\Omega_1,\ldots,\Omega_n](x^\circ)$. *If X is Asplund then*

$$\eta[\Omega_1,\ldots,\Omega_n](x^\circ)\le\frac{\hat{\theta}[\Omega_1,\ldots,\Omega_n](x^\circ)}{[1-\hat{\theta}[\Omega_1,\ldots,\Omega_n](x^\circ)]_+}.$$

Corollary 1. *Let X be Asplund. The collection of sets $\Omega_1,\ \Omega_2,\ \ldots,\ \Omega_n$ is strongly regular (weakly stationary) at x° if and only if $\eta[\Omega_1,\ldots,\Omega_n](x^\circ)>0$ $(=0)$.*

The weak stationarity part of the last statement is related to fundamental concepts of variational analysis. It recaptures in the Asplund space setting the *Extended Extremal Principle* and strengthens the *Extremal Principle.*

Extremal Principle ([6, 7]) *If the collection of sets* $\Omega_1, \Omega_2, \ldots, \Omega_n$ *is locally extremal at* x° *then* $\eta[\Omega_1, \ldots, \Omega_n](x^\circ) = 0$.

Extended Extremal Principle ([2]) *The collection of sets* $\Omega_1, \Omega_2, \ldots, \Omega_n$ *is weakly stationary at* x° *if and only if* $\eta[\Omega_1, \ldots, \Omega_n](x^\circ) = 0$.

Taking into account the extremal characterizations of Asplund spaces in [7] one can formulate the following theorem.

Theorem 3 ([3]) *The following assertions are equivalent:*

1. *X is an Asplund space;*
2. *The Extremal principle is valid in X;*
3. *The Extended extremal principle is valid in X.*

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CONDITIONS OF CONIC EXTREMALITY

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Introduction. The paper is dedicated to the problems of non-scalar optimization, i.e. to the problems of vector optimization, in which a criteria space is not necessarily finite-dimensional. There are two motives of the consideration of such problems: the classical concepts of the solution of the vector optimization problem don't allow the direct generalization in case of infinite-dimensional spaces [1]; there is a connection between the theory of mathematical games and the non-scalar optimization.

1. Basic concepts and facts. The reflexive and transitive relation $\succcurlyeq$ is a linear partial order in the linear space $\mathfrak{W}$, if $a \succcurlyeq b \Rightarrow a + c \succcurlyeq b + c, ta \succcurlyeq tb. \forall a, b, c \in \mathfrak{W}, t > 0$. For the linear partial order $\succcurlyeq$ the set $C = \{a \in \mathfrak{W} | a \succcurlyeq 0\}$ is a convex cone with the vertex $0 \in \mathfrak{W}$. Conversely, the convex cone $C \subset \mathfrak{W}$ with the zero vertex in the linear space $\mathfrak{W}$ induces the linear order $\succcurlyeq$ by the condition $a \succcurlyeq b \Leftrightarrow a - b \in \mathfrak{W}$. The partially ordered linear space $\mathfrak{W}$ with the convex cone $C \subset \mathfrak{W}, 0 \in C$. Below, without a special stipulation, the cone $C \subset \mathfrak{W}$ is considered sharp, i.e. $C \cap (-C) = \{0\}$.