here

$$
\begin{aligned}
f_0(y,u) &= \sum_{i=1}^{m^k}\sum_{j=1}^{m}\left(\sum_{s=1}^{m}\left(c_{j(m)}z\left(t,m^k(i-1)+s^j\right)y\left(t,m^k(i-1)+s^j\right)\right)\right), \\
& k \le t < k+1, \quad \left(k=\overline{0,N}\right).
\end{aligned} \tag{8}
$$

The equations of motions are:

$$
\dot{y}(t,j(k+1)) = \ln u\left[y(t,j(k))+m^k(j(t)-1)+\frac{s_i}{1-k}\right], \quad y(0,1)=x, \tag{9}
$$

$$
\dot{z}(t,j(k+1)) = z(t,j(k+1))\ln p_i, \quad z(0,j)=1; \quad k\le t<k+1. \tag{10}
$$

$$
j(k+1)=j(k)+m^k(i-1), \quad k=0,1,\ldots
$$

$$
y(k,j(k+1))=y(k,j(k)),
$$

Further, we will find optimal control by using maximum principle for (8)-(10) problem.

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REACHABILITY OF CONE FRACTIONAL CONTINUOUS-TIME LINEAR SYSTEMS

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Abstract. A new class of cone fractional continuous-time linear systems is introduced. Necessary and sufficient conditions for the fractional linear systems to be the cone fractional systems are established. Sufficient conditions for the reachability of the cone fractional systems are given. The considerations are illustrated by an example of linear cone fractional systems.

From the author's introduction. In positive systems inputs, state variables and outputs take only non-negative values. Examples of positive systems are industrial processes involving chemical reactors, heat exchangers and distillation columns, storage systems, compartmental systems, water and atmospheric pollution models. A variety of models having positive linear systems behaviour can be found in engineering, management science, economics, social sciences, biology and medicine, etc.

Positive linear systems are defined on cones and not on linear spaces. Therefore, the theory of positive systems is more complicated and less advanced. An extension of positive systems are the cone systems. The notion of cone systems was introduced in [1]. Roughly speaking cone system is a system obtained from positive one by substitution of the positive orthants of states, inputs and outputs by suitable arbitrary cones. The realization problem for cone systems has been addressed in [1, 2].

The first definition of the fractional derivative was introduced by Liouville and Rieman at the end of the 19th century. This idea has been used by engineers for modelling different process in the late 1960s.

In this paper the notion of cone fractional linear continues-time system will be introduced. Sufficient conditions for the reachability of the cone fractional linear systems will be established. To the best knowledge of the author the cone fractional continuous-time linear systems have not been considered yet.

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A THEOREM OF INSTABILITY

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In this communication, we state a generalization of the theorem of N. N. Krassovskii about instability for various classes of semidynamic and dynamic systems. Denoting by E the set $\{x \mid \dot{V}(x) = 0\}$ where the derivative of the Lyapunov's Vfunction vanishes, we present a case where E can contain semitrajectories.

1. Semidynamic system. Let (X, d) be a metric space and $(X, \mathbb{R}^+, \pi)$ a semidynamic system [1]; hereafter the notation xt will stand for $\pi(x, t)$. If $N \subset X$ and $I \subset \mathbb{R}^+$, we denote by NI the set defined by $NI = \{x \in X : \ x \in N, \ t \in I\}$. For every $x \in X$, we assume that the mapping $t \mapsto xt$ is defined for all $t \geq 0$ and we define the positive semitrajectory $\gamma^+(x)$ as $\gamma^+(x) = x\mathbb{R}^+$.

We will say that $N \subset X$ is positively invariant, if $N\mathbb{R}^+ = N$ and weakly invariant, if $\forall x \in N$ and $\forall \tau > 0 \ \exists y \in N$, such that $\gamma^+(y)\tau = x$ and $\gamma^+(y) \subset N$ [1]. The symbol $L^+(x)$ will denote the set of all ω-limit points for $x \in X$, $\varnothing$ denotes the empty set. We denote by $\mathrm{cl}(N)$ and ∂N the closure and the border of set N respectively; $\partial_{Y_0} M$ will denote the border of M in Y_0. For $\alpha > 0$, we denote by $B(N, \alpha)$ the set defined by $B(N, \alpha) = \{x \in X : \ d(N, x) < \alpha\}, \ \alpha > 0$.

Let M and Y be two closed nonempty subsets such, that $M \subset Y$. A set M is said to be unstable, if $(\forall \varepsilon > 0)(\forall \delta > 0)(\exists m \in M)(\exists p \in B(m, \delta))(\exists t^* = t^*(p) > 0) : d(pt^*, M) \geq \varepsilon$; M is said to be a uniform semi-weak attractor relatively to the set Y, if $(\forall \alpha > 0)(\exists \sigma = \sigma(\alpha) > 0)(\forall \delta > 0)(\exists T = T(\sigma) > 0) : \ (B(M, \sigma) \cap Y)T \subset B(M, \delta) \cap Y$ [2].

We will say, that M satisfies the condition (K) if there exists a neighborhood U of M, such that if $pt \in B(M, \mu) \subset U$ for $\mu > 0$ and $t \geq 0$, the semitrajectory $\gamma^+(p)$ is relatively compact. A function $a : \mathbb{R}^+ \to \mathbb{R}^+$ continuous and increasing such that $a(0) = 0$ is called a function of class $\mathcal{K}$.