

A single-valued mapping $u : X \rightarrow Y$ is called a *selection* of a multifunction $\mathcal{A} : X \rightrightarrows Y$ if $u(x) \in \mathcal{A}(x)$ for all $x \in \text{dom}\mathcal{A}$.

A family $\mathfrak{A}$ consisting of selections of a multivalued mapping $\mathcal{A} : X \rightrightarrows Y$ will be said to be *exhaustive* for $\mathcal{A}$ if $\mathcal{A}(x) = \{u(x) \mid u \in \mathfrak{A}\}$ for all $x \in \text{dom}\mathcal{A}$.

In the paper we focus our attention on affine selections of affine multivalued mappings. We prove that for each affine multivalued mapping with compact values there exists an exhaustive family of affine selections and, consequently, it can be represented by its affine selections. Moreover, we show that a convex multifunction with compact values is affine if and only if it possesses an exhaustive family of affine selections. Thus the existence of an exhaustive family of affine selections is the characteristic feature of affine multifunctions which differs them from other convex multifunctions with compact values.

As a subset of the normed vector space of single-valued affine functions from X into Y the family of all affine selections of an affine multifunction with compact values is convex and compact. It is natural to put the converse question: when a given convex compact subset of single-valued affine functions is an exhaustive family of affine selections for some affine multifunction? To show that this question is not trivial we present an example of convex and compact subset of single-valued affine functions from $\mathbb{R}$ to $\mathbb{R}$ that is an exhaustive family of affine selections for the concave multifunction from $\mathbb{R}$ to $\mathbb{R}$ the restriction of which on any nontrivial interval of $\mathbb{R}$ is not affine. In the paper we obtain necessary and sufficient conditions for a convex and compact subset of single-valued affine functions to generate a multifunction that is affine on a given convex subset.

We introduce also the notion of exposed selections as well as the notion of extreme selections of a multifunction and prove that each affine multifunction with compact values can be represented as the closed convex hull of its exposed affine selections and as the convex hull of its extreme affine selections. These statements extend the Straszewicz theorem and the Krein-Milman theorem to affine multifunctions.

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THE MATHEMATICAL MODEL OF INVENTORY CONTROL WITH DISCRETE DEMANDS

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In this paper mathematical model of multistage of inventory control problem with discrete demands is investigated. The inventory control is one of the newest operation researches branches. In 1951 economists Arrow (laureate of the Nobel prize in 1972) formulated the creation principles of the mathematical model for inventory control. Our multistage of inventory control problem is

described by the dynamic programming equations. A new inventory control problem equivalent to the given is constructed. Applying the maximum principle solve this new problem.

Let x be initial quantity of products stores in a warehouse. We also assume, that products are ordering at the discrete moments $t = 0, 1, \dots, n$, the order fulfill at the same moments of the time.

Suppose given functions: $\varphi(s_i) = p_i$ – density function of demands; $c(z)$ – the order's price function (z – quantity of product's ordered); $L(z)$ – penalty function (penalty for shortage of z – product's); $\alpha (0 < \alpha < 1)$ – discount coefficient.

The discrete inventory process is described by the dynamic programming equations, namely:

$$f_n(x) = \min_{y \geq x} \left[c(y - x) + \sum_{s_j > y} p_j L(y) + \alpha \sum_{i=1}^m p_i f_{n-1}(y - s_i) \right], \quad f_0(\cdot) = 0. \quad (1)$$

If $ux = y (u \geq 1, x > 0)$, from the relations (1) we have

$$f_n(x) = \min_{y \geq x} \left[g(u, x) + \alpha \sum_{i=1}^m p_i f_{n-1}(ux - s_i) \right], \quad f_0(\cdot) = 0, \quad (2)$$

when

$$g(u, x) = c(x(u - 1)) + L(ux) \sum_{s_j > ux} p_j.$$

We use a theorem from [2] establishing a connection between dynamic programming and optimal control process. According to this theorem we represent new optimal control process equivalent to dynamic process.

The criterion of quantity for initial process is expressed as

$$I_N(x, \{u_t\}) = \sum_{t=0}^n \sum_{k=1}^{m^t} z_t(k) g(x_t, u_t) \rightarrow \inf. \quad (3)$$

The equations of motion are:

$$x_{t+1}(i(t+1)) = u_t x_i(i(t)) - s_j, \quad x_0(1) = x; \quad (4)$$

$$z_{t+1}(i(t+1)) = z_i(i(t)) p_j, \quad z_0(1) = 1; \quad (5)$$

$$u_t \geq 1, \quad t = \overline{0, N-1}; \quad i(t) \in J(t), \quad J_{t+1} = J_t + (j-1)m^t,$$

$$t = 0, 1, \dots; \quad j = \overline{1, m}, \quad J_0 = 1.$$

Equivalence between process (1)-(2) and (3)-(5) is described by the following relations

$$\inf I(x, \{u_t\}) = f_N(x).$$

We investigate solutions of the problem (3)-(5) is describe by the polynomial form

$$g(x, u) = \sum_{i=0}^n (c_i u^i + b_i) x^i. \quad (6)$$

Now continuous process for the problem (3)-(5) described as follows:

$$\int_0^N f_0(y(t), u(t)) dt \rightarrow \inf, \quad (7)$$

here

$$f_0(y, u) = \sum_{i=1}^{m^k} \sum_{j=1}^m \left(\sum_{s=1}^m \left(c_{j(m)} z \left(t, m^k (i-1) + s^j \right) y \left(t, m^k (i-1) + s^j \right) \right) \right), \quad (8)$$

$$k \leq t < k+1, \quad (k = \overline{0, N}).$$

The equations of motions are:

$$\dot{y}(t, j(k+1)) = \ln u \left[y(t, j(k)) + m^k (j(t) - 1) + \frac{s_i}{1-k} \right], \quad y(0, 1) = x, \quad (9)$$

$$\dot{z}(t, j(k+1)) = z(t, j(k+1)) \ln p_i, \quad z(0, j) = 1; \quad k \leq t < k+1. \quad (10)$$

$$j(k+1) = j(k) + m^k (i-1), \quad k = 0, 1, \dots$$

$$y(k, j(k+1)) = y(k, j(k)),$$

Further, we will find optimal control by using maximum principle for (8)-(10) problem.

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REACHABILITY OF CONE FRACTIONAL CONTINUOUS-TIME LINEAR SYSTEMS

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Abstract. A new class of cone fractional continuous-time linear systems is introduced. Necessary and sufficient conditions for the fractional linear systems to be the cone fractional systems are established. Sufficient conditions for the reachability of the cone fractional systems are given. The considerations are illustrated by an example of linear cone fractional systems.

From the author's introduction. In positive systems inputs, state variables and outputs take only non-negative values. Examples of positive systems are industrial processes involving chemical reactors, heat exchangers and distillation columns, storage systems, compartmental systems, water and atmospheric pollution models. A variety of models having positive linear systems behaviour can be found in engineering, management science, economics, social sciences, biology and medicine, etc.

Positive linear systems are defined on cones and not on linear spaces. Therefore, the theory of positive systems is more complicated and less advanced. An extension of positive systems are the cone systems. The notion of cone systems was introduced in [1]. Roughly speaking cone system is a system obtained from positive one by substitution of the positive orthants of states, inputs and outputs by suitable arbitrary cones. The realization problem for cone systems has been addressed in [1, 2].