Introduce the piecewise continuous functions [5] $V : R_+ \times \Omega \to R$, $V(t,0) = 0$ for $t \in R_+$. Let V is continuously differentiable if $t \neq \tau_i(x)$ and we define the following derivatives of $V(t,x)$:

$$\dot{V}(t,x) = \frac{\partial V}{\partial t}(t,x) + \frac{\partial V}{\partial x}(t,x) f(t,x).$$

Denote $\Delta V_i = V(\tau_i + 0, x + I_i(x)) - V(\tau_i, x), \quad i \in N$. We will assume this functions left continuous.

Using this functions with non-positive derivative, the effective sufficient conditions of asymptotic stability were obtained. The instability theorem was proved using function $V(t,x)$ with non-negative derivative.

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EXTREMAL STRUCTURE OF AFFINE MULTIVALUED MAPPINGS

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Let X and Y be two sets and let $\mathcal{A} : X \rightrightarrows Y$ be a multivalued mapping from X into Y which assigns to each $x \in X$ a (possibly empty) subset $\mathcal{A}(x) \subset Y$.

A multifunction $\mathcal{A} : X \rightrightarrows Y$ is called

(i) *convex* if

$$\alpha\mathcal{A}(x_1) + (1-\alpha)\mathcal{A}(x_2) \subset \mathcal{A}(\alpha x_1 + (1-\alpha)x_2)$$

for all $x_1, x_2 \in \mathrm{dom}\mathcal{A}$ and $\alpha \in [0,1]$;

(ii) *concave* if $\mathrm{dom}A$ is a convex subset of X and

$$\alpha\mathcal{A}(x_1) + (1-\alpha)\mathcal{A}(x_2) \supset \mathcal{A}(\alpha x_1 + (1-\alpha)x_2)$$

for all $x_1, x_2 \in \mathrm{dom}\mathcal{A}$ and $\alpha \in [0,1]$.

A multivalued mapping $\mathcal{A} : X \rightrightarrows Y$ is called *affine* if and only if it is both convex and concave on $\mathrm{dom}\mathcal{A}$, that is, if

$$\alpha\mathcal{A}(x_1) + (1-\alpha)\mathcal{A}(x_2) = \mathcal{A}(\alpha x_1 + (1-\alpha)x_2)$$

for all $x_1, x_2 \in \mathrm{dom}\mathcal{A}$ and $\alpha \in [0,1]$.

Throughout the paper we will suppose that values of multivalued mappings involved in the study are convex and compact subsets of Y.

A single-valued mapping $u : X \to Y$ is called *a selection* of a mulifunction $\mathcal{A} : X \rightrightarrows Y$ if $u(x) \in \mathcal{A}(x)$ for all $x \in \mathrm{dom}\mathcal{A}$.

A family $\mathfrak{A}$ consisting of selections of a multivalued mapping $\mathcal{A} : X \rightrightarrows Y$ will be said to be *exhaustive* for $\mathcal{A}$ if $\mathcal{A}(x) = \{u(x) \,|\, u \in \mathfrak{A}\}$ for all $x \in \mathrm{dom}\mathcal{A}$.

In the paper we focus our attention on affine selections of affine multivalued mappings. We prove that for each affine multivalued mapping with compact values there exists an exhaustive family of affine selections and, consequently, it can be represented by its affine selections. Moreover, we show that a convex multifunction with compact values is affine if and only if it possesses an exhaustive family of affine selections. Thus the existence of an exhaustive family of affine selections is the characteristic feature of affine multifunctions which differs them from other convex multifunctions with compact values.

As a subset of the normed vector space of single-valued affine functions from X into Y the family of all affine selections of an affine multifunction with compact values is convex and compact. It is natural to put the converse question: when a given convex compact subset of single-valued affine functions is an exhaustive family of affine selections for some affine multifunction? To show that this question is not trivial we present an example of convex and compact subset of single-valued affine functions from $\mathbb{R}$ to $\mathbb{R}$ that is an exhaustive family of affine selections for the concave multifunction from $\mathbb{R}$ to $\mathbb{R}$ the restriction of which on any nontrivial interval of $\mathbb{R}$ is not affine. In the paper we obtain necessary and sufficient conditions for a convex and compact subset of single-valued affine functions to generate a multifunction that is affine on a given convex subset.

We introduce also the notion of exposed selections as well as the notion of extreme selections of a multifunction and prove that each affine multifunction with compact values can be represented as the closed convex hull of its exposed affine selections and as the convex hull of its extreme affine selections. These statements extend the Straszewicz theorem and the Krein-Milman theorem to affine multifunctions.

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THE MATHEMATICAL MODEL OF INVENTORY CONTROL WITH DISCRETE DEMANDS

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In this paper mathematical model of multistage of inventory control problem with discrete demands is investigated. The inventory control is one of the newest operation researches branches. In 1951 economists Arrow (laureate of the Nobel prize in 1972) formulated the creation principles of the mathematical model for inventory control. Our multistage of inventory control problem is