ELLIPSOIDAL TECHNIQUES IN STATE ESTIMATION PROBLEMS FOR NONLINEAR CONTROL SYSTEMS UNDER UNCERTAINTY

T.F. Filippova

Institute of Mathematics and Mechanics, Russian Academy of Sciences
16 S. Kovalevskaya str., 620219 Ekaterinburg, Russia
ftf@imm.uran.ru

Introduction. The paper is devoted to state estimation problems [1, 2, 3, 4] for nonlinear uncertain dynamic systems with system states being compact sets. Applying results of ellipsoidal estimation theory developed for linear control systems we present new approaches that allow to find set-valued estimates for reachable sets of uncertain nonlinear control systems. Numerical simulations are also given.

1. Problem Formulation. The paper deals with the problems of control and state estimation for a dynamical control system

$$\dot{x}(t) = A(t)x(t) + f(x(t)) + G(t)u(t), \ x \in R^n, \ t_0 \le t \le T, \tag{1}$$

with unknown but bounded initial condition

$$x(t_0) = x_0, \quad x_0 \in X_0, \quad X_0 \subset R^n, \tag{2}$$

$$u(t) \in U, \quad U \subset R^m, \quad \text{for a.e. } t \in [t_0, T]. \tag{3}$$

Here matrices $A(t)$ and $G(t)$ (of dimensions $n \times n$ and $n \times m$, respectively) are assumed to be continuous on $t \in [t_0, T]$, X_0 and U are compact and convex. The nonlinear n-vector function $f(x)$ in (1) is assumed to be of quadratic type

$$f(x) = (f_1(x), \ldots, f_n(x)), \ f_i(x) = x'B_i x, \ i = 1, \ldots, n, \tag{4}$$

where B_i is a constant $n \times n$ - matrix ($i = 1, \ldots, n$).

Consider the following differential inclusion [5] related to (1)–(3)

$$\dot{x}(t) \in A(t)x(t) + f(x(t)) + P(t), \quad \text{for a.e. } t \in [t_0, T], \ x(t_0) = x_0 \in X_0, \tag{5}$$

where $P(t) = G(t)U$.

Let absolutely continuous function $x(t) = x(t, t_0, x_0)$ be a solution to (5) with initial state x_0 satisfying (2). The differential system (1)–(3) (or equivalently, (5)) is studied here in the framework of the theory of uncertain dynamical systems (differential inclusions) through the techniques of trajectory tubes [2]

$$X(\cdot) = \bigcup_{x_0 \in X_0} \{x(\cdot) = x(\cdot, t_0, x_0)\} \tag{6}$$

combining all solutions $x(\cdot, t_0, x_0)$ to (5).

The problem consists in describing and estimating the trajectory tube $X(\cdot)$ of the differential inclusion (5). The point of special interest is to find the t – cross-section $X(t)$ of this map that is actually the attainability domain (reachable set) of the system (1)–(3) at the instant t or to construct its estimates.

2. Main Results. The modified state estimation approaches which use the special structure of nonlinearity of studied control system (1)–(4) and advantages of ellipsoidal calculus [3, 4] are developed.

We consider here new ellipsoidal techniques related to construction of external and internal estimates of reachable sets and trajectory tubes of the nonlinear system. Some estimation algorithms basing on combination of discrete-time versions of funnel equations [6, 2] and ellipsoidal calculus [3, 4] are given.

Examples and numerical results related to procedures of set-valued approximations of trajectory tubes and reachable sets are also presented [7]. The applications of the problems studied in this paper are in guaranteed state estimation for nonlinear systems with unknown but bounded errors and in nonlinear control theory.

The research was supported by the Russian Foundation for Basic Research (RFBR) under the Project No. 06-01-00483.

References

1. *Kurzhanski A.B.* Control and Observation under Conditions of Uncertainty. Nauka, Moscow, 1977.
2. *Kurzhanski A.B., Filippova T.F.* On the theory of trajectory tubes — a mathematical formalism for uncertain dynamics, viability and control // Advances in Nonlinear Dynamics and Control: a Report from Russia, Progress in Systems and Control Theory, A.B. Kurzhanski, eds. Birkhauser, Boston, 1993. P. 22–188.
3. *Kurzhanski A.B., Valyi I.* Ellipsoidal Calculus for Estimation and Control. Birkhauser, Boston, 1997.
4. *Chernousko F.L.* State Estimation for Dynamic Systems. Nauka, Moscow, 1988.
5. *Filippov A.F.* Differential Equations with Discontinuous Right-hand Side. Nauka, Moscow, 1985.
6. *Panasyuk A.I.* Equations of attainable set dynamics, Part 1: Integral funnel equations // J. Optimiz. Theory Appl. 1990. V. 64. No. 2. P. 349–366.
7. *Filippova T.F., Berezina E.V.* On state estimation approaches for uncertain dynamical systems with quadratic nonlinearity: theory and computer simulations // Proc. 6th Int. Conf. Large-Scale Scientific Computations LSSC'07. Sozopol, Bulgaria, 2007. P. 20–21.

ON ASYMPTOTIC STABILITY OF THE DYNAMICAL SYSTEMS WITH DISCONTINUOUS SOLUTIONS

R.I. Gladilina

Donetsk National Technical University, 58 Artem str., 83000 Donetsk, Ukraine
rgladilina@yandex.ru

The present paper deals with dynamical systems having discontinuous trajectories. This systems may be successfully describe by means of the systems of differential equations with impulse effect [1].

Using the ideas of Barbashin-Krasovsky [2] to the stability analysis of the periodic impulsive systems, in [3,4] the sufficient conditions of asymptotic stability and instability of the zero solutions were obtained with the aid of the second Lyapunov method.

We will discuss the stability problem of the trivial solution of not periodic impulsive systems of the most general type: with discontinuities at unfixed instants of time

$$\begin{aligned} \frac{dx}{dt} &= f(t,x), \quad t \neq \tau_i(x), \\ \Delta x &= I_i(x), \quad t = \tau_i(x), \quad i \in N, \\ x(t_0) &= x_0, \end{aligned}$$

where $t \in R_+,\ x \in \Omega \subset R^n,\ f : R_+ \times \Omega \to R^n,\ f(t,0) \equiv 0;\ I_i : \Omega \to R^n,\ I_i(0) \equiv 0,$ $\tau_i : \Omega \to R_+,\ 0 < \tau_1(x) < \tau_2(x) < \ldots$ and $\tau_i(x) \to \infty$ if $i \to \infty$.

Let suppose that solution $x = x(t)$ of the system is left continuous.