DUALITY THEOREM FOR LINEAR DISCRETE–TIME FRACTIONAL CONTROL SYSTEMS

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In the paper we deal with linear discrete-time fractional control systems in generalized form for sampling interval $c > 0$. We give definition of observability of such systems and discuss possible dual forms corresponding to property of controllability in q steps. We formulate and prove theorem of duality. We discuss the stability of rank conditions for two–dimensional linear system. The stability property is under the investigations in dependence of an generalized order of a fractional system.

STABILITY OF CANONICAL PERIODIC MATRIX IMPULSIVE DIFFERENTIAL EQUATIONS

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In our report we consider canonical periodic matrix impulsive differential equation

$$dZ/dt - i\mathcal{J}\mathcal{A}(t)Z = 0,\ t \neq t_j;\ \triangle(Z) = i\mathcal{J}\mathcal{D}_jX,\ t = t_j, \tag{1}$$

where i is complex identity, $Z \in C_2^{n\times m}$, $Z = (X,Y)^T$, $X, Y \in C^{n\times m}$, $\mathcal{J} = \mathcal{J}^*$, $\mathcal{J}^{-1} = \mathcal{J}$, $\mathcal{J} = \mathcal{P}_1 - \mathcal{P}_2$, $\mathcal{P}_i$ are projection operator in $C_2^{n\times m}$, $\mathcal{P}_1Z = X$, $\mathcal{P}_2Z = Y$, $\mathcal{A} = \begin{pmatrix} [A_{11}] & [A_{12}] \\ [A_{21}] & [A_{22}] \end{pmatrix}$, $\mathcal{D}_j = \begin{pmatrix} 0 & 0 \\ 0 & -[D_j] \end{pmatrix}$, $\mathcal{A}^*(t) = \mathcal{A}(t)$, $[D_j]Y = D_jY\widetilde{D}_j$, $[A_{i1}]X = A_{i1}X\widetilde{A}_{i1}$, $A_{ij}, D_j \in C^{n\times n}$, $\widetilde{A}_{ij}, \widetilde{D}_j \in C^{m\times m}$, $C^{n\times m}$ is the space of complex $n \times m$ matrices, $\|X\| = \sqrt{Tr(X_1^*X_1) + Tr(X_2^*X_2)}$. The equation (1) may be rewritten as impulsive equation [1] in double phase space $C_2^{n\times m} = C^{n\times m} \oplus C^{n\times m}$

$$dZ/dt = i\mathcal{J}\big(\mathcal{A}(t) + \sum_j \mathcal{D}_j\delta(t - t_j)\big)Z, \tag{2}$$

In more general case $\mathcal{J} = sign\mathcal{W} = \mathcal{W}\,|\mathcal{W}|^{-1}$, $|\mathcal{W}| = (\mathcal{W}^*W)^(1/2)$. In real Hilbert space $\mathcal{H}^{(2)} = \mathcal{H} \oplus \mathcal{H}$ the role of operator $(i\mathcal{J})$ play operator $\mathcal{J}_\Gamma = \begin{pmatrix} 0 & [I] \\ -[I] & 0 \end{pmatrix}$, so-called simplectic identity in real double Hilbert space $\mathcal{H}_2$. The equation (2) than is named Hamiltonian equation.

The monodromy operator $\mathcal{U}(T)$ of e equation (1) is J-unitary, i.e.

$$\mathcal{U}^*(T)\mathcal{J}\mathcal{U}(T) = \mathcal{J}. \tag{3}$$

The stability of equation (1) means that the monodromy operator is stable [2].

Theorem 1. *For the equation (1) to be stable necessary and sufficient that the double Hilbert space $\mathcal{H}^{(2)}$ be decomposed to $\mathcal{J}$-orthogonal subspaces $\mathcal{H}_1$ and $\mathcal{H}_2$; $\mathcal{H}^{(2)} = \mathcal{H}_1 \oplus \mathcal{H}_2$, which are invariant for the monodromy operator $\mathcal{U}(T)$ and subspace $\mathcal{H}_1$ be $\mathcal{J}$-positive, subspace $\mathcal{H}_2$ be $\mathcal{J}$-negative.*

Corollary 1. *If the canonical periodic matrix impulsive equation is stable than it is reducible.*

References

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ON THE STABILITY OF INVARIANT SETS OF FUNCTIONAL DIFFERENTIAL EQUATIONS WITH DELAY

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Introduction. Let $t \in \mathbb{R}_+ = [0,\infty)$, $x = (x^1,\dots,x^n) \in \mathbb{R}^n, |x| = \sqrt{\sum_{i=1}^n (x^i)^2}, y = (y^1,\dots,y^m)$, $|y| = \sqrt{\sum_{s=1}^m (y^s)^2}$, $z = (x,y) = (z^1,\dots,z^{n+m}) \in \mathbb{R}^{n+m}$, $|z| = \sqrt{|x|^2+|y|^2}$. For a given $h>0$, C^n and C^m denote the spaces of continuous functions mapping $[-h,0]$ into $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively. Let $\varphi = (\varphi^1,\varphi^2,\dots,\varphi^{n+m}) = (\psi,\lambda)$, where $\psi = (\psi^1,\dots,\psi^n) \in C^n$, $\lambda = (\lambda^1,\dots,\lambda^m) \in C^m$, $C = C^n \times C^m$. Denote

$$\|\psi\| = \sup(|\psi^i(\theta)|, \text{ under } -h \le \theta \le 0,\ 1 \le i \le n),$$

$$\|\lambda\| = \sup(|\lambda^j(\theta)|, \text{ under } -h \le \theta \le 0,\ 1 \le j \le m),$$

$$\|\varphi\| = \max(\|\psi\|, \|\lambda\|),$$

$$C_H = \{\varphi \in C : \|\psi\| \le H,\ \|\lambda\| < +\infty\}.$$

If z is a continuous function of u defined on $-h \le u < A, A>0$, and if t is a fixed number satisfying $0 \le t < A$, then z_t denotes the restriction of z to the segment $[t-h,t]$ so that $z_t = (z_t^1,\dots,z_t^{n+m}) = (x_t,y_t)$ is an element of C defined by $z_t(\theta) = z(t+\theta)$ for $-h \le \theta \le 0$.

Consider a system of functional differential equations

$$\frac{dz(t)}{dt} = Z(t,z_t). \tag{1}$$

In this system dz/dt denotes the right-hand derivative of z at t, t is time, and $Z(t,\varphi) = (X(t,\varphi), Y(t,\varphi)) \in \mathbb{R}^{n+m}$ is defined on $\mathbb{R}_+ \times C_{H_1}$; $X \in \mathbb{R}^n, Y \in \mathbb{R}^m$.

According to T.Burton [1], we denote by $z(t_0,\varphi) = (x(t_0,\varphi), y(t_0,\varphi))$ a solution of (1) with initial condition $\varphi \in C_{H_1}$, where $z_{t_0}(t_0,\varphi) = \varphi$ and we denote by $z(t,t_0,\varphi)$ the value of $z(t_0,\varphi)$ at t and $z_t(t_0,\varphi) = z(t+\theta,t_0,\varphi)$, $-h \le \theta \le 0$.

It is assumed that the vector-valued functional $Z(t,\varphi)$ is continuous on $\mathbb{R}_+ \times C_{H_1}$ so that a solution will exist for each continuous initial condition. We suppose that each solution $z(t_0,\varphi)$ is defined for those $t \ge t_0$, such that $\|x_t(t_0,\varphi)\| < H_1$.

Let $V(t,\varphi)$ be a continuous functional defined for $t \ge 0, \varphi \in C_{H_1}$.

Consider the set

$$M := \{\varphi \in C : \|\psi\| = 0, \|\lambda\| < \infty\}. \tag{2}$$

The necessary and sufficient conditions of the uniform asymptotic stability of the invariant set M of system (1) were obtained in [2]. In that paper, the method of Lyapunov functionals, founded