NUMERICAL INVESTIGATION OF OPTIMIZATION ASPECTS IN OPTIMAL CONTROL PROBLEMS TO SOME PARTIAL DIFFERENTIAL SYSTEMS

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Introduction. In our paper we consider the results of numerical experience to optimal synthesis of parabolic differential systems. We investigate a times quantity to improve a quality criteria of optimal controls problem. We investigate a times quantity to integrate partial differential systems also. Our algorithms are based from approaches has been suggested by R. Gabasov and F.M. Kirillova [1].

1. Problem formulation to 2D system. We consider the following parabolic system (1):

$$\begin{cases} \frac{\partial\varphi(t,x)}{\partial t} = L_x\varphi(t,x) + \omega(t), & x \in (x_0, x^*),\ t \in (t_0, t^*); \\ A \cdot \frac{\partial\varphi}{\partial x}(t, x^*) = \varphi(t, x_0) = u(t); & t \in (t_0, t^*];\ \varphi(t_0, x) = 0;\ x \in [x_0, x^*]; \end{cases} \quad (1)$$

Here:

$$L_x\varphi(t,x) = \frac{\partial}{\partial x}(A \cdot \frac{\partial\varphi}{\partial x}) + B \cdot \frac{\partial\varphi}{\partial x} + C \cdot \varphi,\ x \in (x_0, x^*) \quad (2)$$

- linear differential operator.

The components of system (1) - uncertain disturbances function $\omega(t)$, control $u(t)$, system state $\varphi(t,x)$ - are also constrained (3):

$$d_* \le u(t) \le d^*\ t \in T;\ b_* \le \varphi(t,x) \le b^*\ (t,x) \in \Omega;\ \omega_* \le \omega(t) \le \omega^*\ t \in T \quad (3)$$

We consider [2],[3] the following feedback optimal control problem (4):

$$maximize\ J(u) = \int_{t_0}^{t^*} (u(\varphi(t, x^*))dt = \int_{t_0}^{t^*} u(t)dt\ subject\ to\ (1) - (3). \quad (4)$$

2. Problem formulation to 3D system. We consider the following parabolic system (5):

$$\frac{\partial\varphi}{\partial t} = L_{xy}\varphi(t,x,y) = u(t,x,y) + \omega(t,x,y),\ (x,y) \in \Omega,\ t \in T;\ N_{xy}\varphi(x,y) = 0\ (x,y) \in \partial\Omega \quad (5)$$

$$L_{xy}\varphi(x,y) = \frac{\partial}{\partial x}(A\frac{\partial\varphi}{\partial x}) + \frac{\partial}{\partial y}(B\frac{\partial\varphi}{\partial y}) + C\frac{\partial\varphi}{\partial x} + D\frac{\partial\varphi}{\partial y} + E\varphi,\ (x,y) \in \Omega \quad (6)$$

$$N_{xy}\varphi(x,y) = A\frac{\partial\varphi}{\partial x}\cos(n,x) + B\frac{\partial\varphi}{\partial x}\cos(n,y),\ (x,y) \in \partial\Omega \quad (7)$$

The components of system (5) - uncertain disturbances function $\omega(t,x,y)$, control $u(t,x,y)$, system state $\varphi(t,x,y)$ - are also constrained (8):

$$\begin{gathered} \int_T (\int_{\Omega_k} \varphi(t,x,y)dxdy)dt = b_k, k \in K; \Omega_k \subset \Omega; K = \{1, 2, \ldots, \overline{k}\} \\ d_* \le u(t,x,y) \le d^*; t \in T; \omega_* \le \omega \le \omega^*; t \in T \end{gathered} \quad (8)$$

We consider [4] the following feedback optimal control problem:

$$maximize\ J(u) = \int_T (\int_{\Omega_k} u(t,x,y)dxdy)dt\ subject\ to\ (5) - (8). \quad (9)$$

Table 1: Experince results to 2D system

h	N	k	ω	T_{int}	T
0.5	20	3	2	0.0315	0.039
0.2	50	3	2	0.094	0.12
0.05	200	3	2	0.422	0.571
0.5	20	3	4	0.032	0.041
0.2	50	3	4	0.078	0.132
0.05	200	3	4	0.64	0.604

Table 2: Experince results to 3D system

h	N	k	ω	T_{int}	T
0.02	20	3	2	0.039	0.79
0.01	50	3	2	0.088	2.94
0.0067	200	3	2	0.153	7.65
0.02	20	3	4	0.04	0.82
0.01	50	3	4	0.096	3.15
0.0067	200	3	4	0.196	7.98

3. Numerical experience results. A table 1 contain an experience results to optimal controls problem (1)-(4) to 2D differential system. A table 2 contain an experience results to optimal controls problem (5)-(9) to 3D differential system. An experience is carried out by using of MatLab 7 environments , Intel Core2Duo E6420, 2 Gb RAM, Windows XP.

Here: h - time discretizations step, N - time steps quantity, k - quantity of restrictions, ω - limits of disturbances (% to relation of control u), T_{int} - average time of differential systems integration, T - average time to construct feedback optimal controls

4. Conclusion. A large machine resource was spent for solving a partial differential systems to optimal control problems with small dimensions. About 80% machine resources was needed for integration partial differential equations to control problem (1)-(4). Essential machine resource was spent to increasing an optimality criteria for optimal control problems with large dimensions. Only about 2%-5% machine resources was needed for integration partial differential equations to control problem (5)-(9). So, it is very actual development a new and more effective algorithms of optimizations.

References

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