RESEARCH OF THE AVERAGED ESTIMATION OF MIXED MOMENT OF THIRD ORDER AND ITS USE IN CARDIOLIGICAL DATA ANALYSIS

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Abstract

We consider here statistical properties of the averaged estimation of mixed moment of third order of stationary stochastic processes. Mathematical expectations, covariance, dispersion of this estimation are calculated. The averaged estimation of mixed moments of third order are unbiased and consistent in root-mean-square sense, have normal limit distribution. Demonstrated, that investigated estimation have good application to cardiological data.

1 Introduction

The Statistical analysis of the time series is one of quickly developing and significant in applied and theoretical relations by direction theory of chances and mathematical statistics. This article is dedicated to building and study statistical characteristic averaged estimation of the mixed moment of the third order.

Let's examine the real stationary stochastic process $x(t), t \in Z$. Suppose that $Mx(t) = 0, t \in Z$. Let, there is N consequent through equal gap of the observations $x(0), x(1), ..., x(N-1)$ for process$x(t), t \in Z$. We shall expect that number of the observations N is enough great. In this case using the estimations of the mixed moments of 3-d order for study applied given by means of software is not comfortable since such study occupies much time. For this we shell separate the whole sample of importance on m parts. $K = \frac{N}{m}$- is the size of sample in every m intervals. K is integer, in other case, we will delete the end or the beginning of the samples.

2 Statistical properties of the averaged estimation of mixed moment of the third order

The averaged estimation of the mixed moment of the third order we will write the following way:

$$\begin{gathered}\widehat{m}_3(t_1,t_2) = \frac{1}{m}\sum_{i=1}^{m}\frac{1}{K}\sum_{t=(i-1)K}^{iK-1} x(t_1+t)x(t_2+t)x(t) = \\ = \frac{1}{N}\sum_{i=1}^{m}\sum_{t=(i-1)K}^{iK-1} x(t_1+t)x(t_2+t)x(t), t, t_1, t_2 \in N\end{gathered} \tag{1}$$

Theorem 1. *For the averaged estimation* (1)
$M\widehat{m}_3(t_1,t_2) = m_3(t_1,t_2)$, *where* $t_1, t_2 \in Z$.

Proof of the theorem is possible to find in work [2]. Thus, the constructed averaged estimation is unbiased estimation.

Theorem 2. *For the averaged estimation of mixed moment of the third order:*

$$cov\left\{\widehat{m}_3\left(i_1,j_1\right), \widehat{m}_3\left(i_2,j_2\right)\right\} = \frac{2\pi}{N}\int\limits_{\Pi} \Phi_N\left(z\right)G(z)dz, \tag{2}$$

$$G(z) = [\underset{\Pi^4}{\int\ldots\int} f_6\left(z-x_2-x_3,x_2,x_3,x_4,x_5\right)\times$$
$$\times e^{i[i_1x_2+x_3j_1+x_4i_2+x_5j_2]}dx_2dx_3dx_4dx_5\left\{1\right\}+$$
$$+\iiint\limits_{\Pi^3} f\left(x\right)f_4\left(z-x-y_2,y_2,y_3\right)e^{i[(z-x-y_2)i_1+y_3i_2+y_2j_1-xj_2]}dxdy_2dy_3\left\{15\right\}+$$
$$+\iiint\limits_{\Pi^3} f_3\left(z-x_2-y_1,x_2\right)f_3\left(y_1,y_2\right)e^{i[-(z-x_2-y_1)j_2+x_2(i_1-j_2)+y_1j_1+y_2i_2]}dx_2dy_1dy_2\left\{9\right\}+$$
$$+\iint\limits_{\Pi^2} f\left(x_1\right)f\left(z-x_1-x_3\right)f\left(x_3\right)e^{i[-x_1i_2+(z-x_1-x_3)(i_1-j_2)+x_3j_1]}dx_1dx_3\left\{15\right\}].$$

Proof of the theorem is possible to find in work [2].

Theorem 3. *Let cumulant spectral density of the k-th order* $f_k(x_1,\ldots,x_{k-1}), x_j \in \Pi, j=\overline{1,k-1}, k=\overline{2,4}$, *unceasing on* Π^{k-1}, *then*
$\lim\limits_{N\to\infty} cov\left\{\widehat{m}_3\left(i_1,j_1\right), \widehat{m}_3\left(i_2,j_2\right)\right\} = 0$, *for* $i_1,i_2,j_1,j_2 \in Z$.

Proof of the theorem is possible to find in work [2].
Consequence 1. The theorem 3 when $i_1 = i_2, j_1 = j_2$ *implies*
$\lim\limits_{N\to\infty} D\widehat{m}_3(i_1,j_1) = 0$,*for* $i_1, j_1 \in Z$.
Thus, built averaged estimation of the mixed moment of the third order is consistent in root-mean-square meaning. Using cumulant approach we find limited distribution of the observable estimates of the mixed moment of the 3-d order.

Theorem 4. *If the ratio realized*

$$\sum_{u_1=-\infty}^{\infty}\ldots\sum_{u_{n-1}=-\infty}^{\infty}\left|cum\left\{x(u_i+\tau_{ij});(i,j)\in\nu_s\right\}\right|<\infty, \tag{3}$$

$s=\overline{1,p}$, $\quad\nu_1,\ldots,\nu_p-$*is indecomposable decomposition,* $\nu=\nu_1\cup\ldots\cup\nu_p$ *then*

$$cum\left\{\widehat{m}_3\left(l_1,j_1\right),\ldots,\widehat{m}_3\left(l_n,j_n\right)\right\}\underset{T\to\infty}{\longrightarrow}0,$$

for $n>2$ *where* $i=\overline{1,n}, j=\overline{1,4}, l_i,j_i,k_i\in Z, i=\overline{1,n}$.

Proof of the theorem repeats proof similar theorem in [1].

Theorem 5. *Under the conditions of theorems 2 and 4 the averaged estimate of the mixed moment of the 3-d order* $\widehat{m}_3\left(l,j\right)$ *has asymptotic normal distribution with expectation value* $m_3\left(l,j\right)$ *and limited covariance structure, corresponding ratio(2).*

3 Application of the investigated estimation to the analysis of cardiological data

One of the courses where application of analysis of stationary stochastic processes is possible, it is cardiology. Data, representing R-R intervals, registered with the help of ECG beforehand are processed to avoid artifacts.

Example 1. Time series 6449 is a norm. 15 years old. In one cardiogram the number of data amounts 36000 meanings. Data of such size are not calculated on the computer. View the sample, consisting of 800 meanings and construct for it the averaged estimation of the moment of 3-d order. We divide sample on 8 intervals, so we get $K = 100, m = 8, N = 800$.

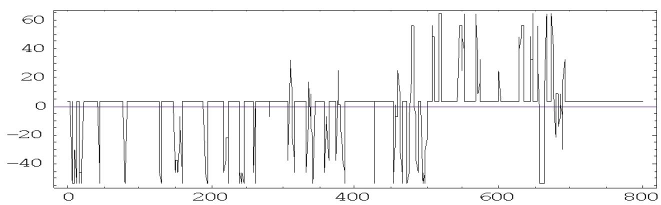

Fig 1. Diagram of basic data

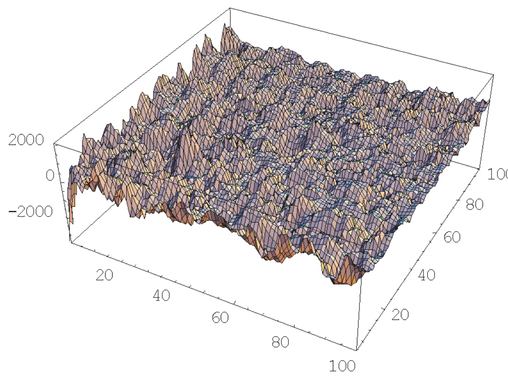	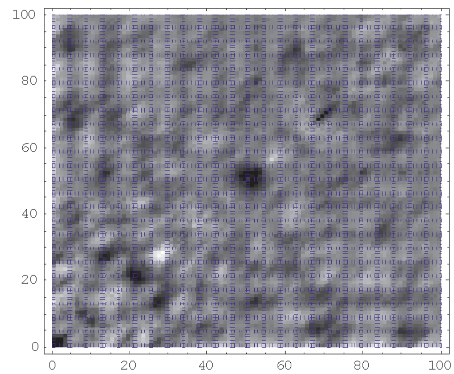
Diagram of the averaged estimation of mixed moment of the third order	Diagram of density of the averaged estimation of mixed moment of the third order

Fig. 2 Analysis of the averaged estimation of mixed moment of the third order

Sequence of basic data has the central tendency equals 638.912. The bar chart is monomodal. Most reveals period is equal 10. Expectation value: Mx= 80.6463, variance: Dx=33153, excess: Es= 2.25607, asymmetry: As= -0.0692034, left-side asymmetry, peaked distribution.

Example 2. Time series 6390 is a norm. 22 years old.

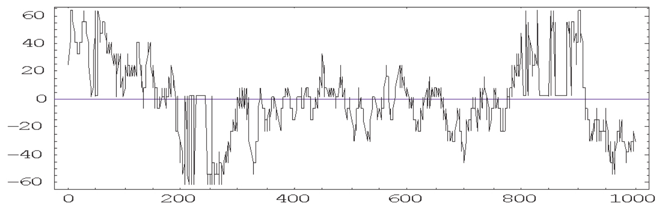

Fig.3 Diagram of basic data.

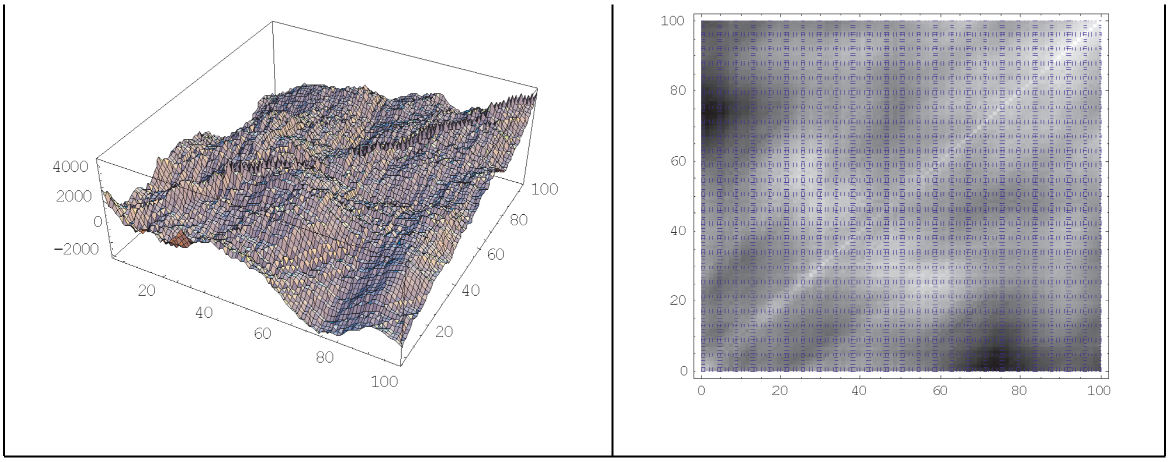

Fig. 4 Analysis of the averaged estimation of mixed moment of the third order.

Most reveals period is equal 7. Expectation value: Mx=1553.27, variance: Dx= 1.20737*10^6, excess: Es= 0.520301, asymmetry: As= -0.627657, left-side asymmetry, peaked distribution.

As the result of investigation of the cardiological data, namely R-R intervals, the mechanisms of changing in the heart rate, connected with patients' age, were evolved. This mechanisms lead to the changing of the view of the diagram of estimation. The estimation of mature people become more smooth and the order of depression decreases. Comparing and studying the local extremum we can notice that for people in the age of 20-40 years, the period divisible 6 beats evolves. And for people in the age of 50-60 years old this period amounts 8-10 beats.

References

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