ANALYSIS OF INCOMES BEHAVIOR IN EXPONENTIAL NETWORKS

M.A. Matalytski, A.V. Pankov
Grodno State University
Grodno, BELARUS
e-mail: a.pankov@gmail.com

Abstract

Exponential queueing networks (QN) of arbitrary architecture with incomes were considered. The behavior of network's systems expected incomes on large time frames was investigated.

1 Introduction

Let's consider open exponential queueing network (QN) with singletype messages. QN consists of n queueing systems (QS) $S_1, S_2, \dots, S_n$. State of the network is described by the vector $k(t) = (k, t) = (k_1, k_2, \dots, k_n, t)$, where k_i – number of messages in system S_i at time moment t, $i = \overline{1, n}$. Stationary Poisson arrival process of messages with rate λ enters the network. Let $\mu_i(k_i)$ be service rate of messages by system S_i, when there are k_i messages in QS S_i, $i = \overline{1, n}$; p_{0j} be probability of message entering from environment to system S_j, $\sum\limits_{j=1}^{n} p_{0j} = 1$; p_{ij} be probability of served message passage from QS S_i to S_j, $\sum\limits_{j=1}^{n} p_{ij} = 1$, $i = \overline{1, n}$; I_i be n-vector with zero components except of ith which equals to 1; $u(x)$ be Heaviside function. It is obvious that $k(t)$ is Markov chain with continuous time and denumerable number of states. When message passes from one system to another, it brings a profit to the second QS, in accordance with it an income of first QS decreases by value of this profit. Such QN can be used as model of incomes changing in banking networks. It can be used also for some logistic and industrial systems investigation. Let's mention that simplest Markov chains with discrete time and incomes were examined in [1] and more complex were examined in [2].

2 Set of difference-differential equations for incomes

Let $v_i(k, t)$ be total expected income, which system S_i obtains during time t, if at initial moment QN is in state k, $r_i(k)$ be income of QS S_i in conventional unit for time unit when QN is in state k, $r_{0i}(k + I_i, t)$ be income of QS S_i, when QN passes from state (k, t) to state $(k + I_i, t + \Delta t)$ during time Δt, $-R_{i0}(k - I_i, t)$ be income of QS S_i, if the network passes from state (k, t) to state $(k - I_i, t + \Delta t)$, $r_{ij}(k + I_i - I_j, t)$ be income of system S_i (expenses or loss of QS S_j), when network changes its state (k,t) by $(k+I_i-I_j, t+\Delta t)$ during time Δt, $i,j=\overline{1,n}$. Possible transitions between network states, their probabilities and incomes of network systems from passages between network states are summarized into table 1.

Table 1: Possible transitions between network states, probabilities and incomes

Possible transitions between network states	Probabilities of transitions	System's S_i incomes for passages between states, $i=\overline{1,n}$
$(k,t)\to(k,t+\Delta t)$	$1-\sum\limits_{j=1}^{n}(\lambda+\mu_j(k_j)u(k_j))\Delta t+$ $+o(\Delta t)$	$r_i(k)\Delta t+v_i(k,t)$
$(k,t)\to(k+I_j,t+\Delta t)$, $j=\overline{1,n}$, $j\neq i$	$\lambda p_{0j}\Delta t+o(\Delta t)$	$r_i(k)\Delta t+v_i(k+I_j,t)$
$(k,t)\to(k-I_j,t+\Delta t)$, $j=\overline{1,n}$, $j\neq i$	$\mu_j(k_j)u(k_j)p_{j0}\Delta t+o(\Delta t)$	$r_i(k)\Delta t+v_i(k+I_j,t)$
$(k,t)\to$ $(k+I_c-I_s,t+\Delta t)$, $c,s=\overline{1,n}$, $c,s\neq i$	$\mu_s(k_s)u(k_s)p_{sc}\Delta t+o(\Delta t)$	$r_i(k)\Delta t+v_i(k+I_c-I_s,t)$
$(k,t)\to(k+I_i,t+\Delta t)$	$\lambda p_{0i}\Delta t+o(\Delta t)$	$r_{0i}(k+I_i,t)+v_i(k+I_i,t)$
$(k,t)\to(k-I_i,t+\Delta t)$	$\mu_i(k_i)u(k_i)p_{i0}\Delta t+o(\Delta t)$	$-R_{i0}(k-I_i,t)+v_i(k-I_i,t)$
$(k,t)\to$ $(k+I_i-I_j,t+\Delta t)$, $j=\overline{1,n}$, $j\neq i$	$\mu_j(k_j)u(k_j)p_{ji}\Delta t+o(\Delta t)$	$r_{ij}(k+I_i-I_j,t)+$ $+v_i(k+I_i-I_j,t)$
$(k,t)\to$ $(k-I_i+I_j,t+\Delta t)$, $j=\overline{1,n}$, $j\neq i$	$\mu_i(k_i)u(k_i)p_{ij}\Delta t+o(\Delta t)$	$-r_{ji}(k-I_i+I_j,t)+$ $+v_i(k-I_i+I_j,t)$

Then it is possible to obtain a set of difference-differential equations (DDE) for expected incomes of QS S_i:

$$\frac{dv_i(k,t)}{dt}=-\sum_{j=1}^{n}\left[\lambda+\mu_j(k_j)u(k_j)\right]v_i(k,t)+$$

$$+\sum_{j=1}^{n}\left[\lambda p_{0j}v_i(k+I_j,t)+\mu_j(k_j)u(k_j)p_{j0}v_i(k-I_j,t)\right]+$$

$$+\sum_{j=1,j\neq i}^{n}\left[\mu_j(k_j)u(k_j)p_{ji}v_i(k+I_i-I_j,t)+\mu_i(k_i)u(k_i)p_{ij}v_i(k-I_i+I_j,t)\right]+$$

$$+\sum_{c,s=1,c,s\neq i}^{n}\mu_s(k_s)p_{sc}v_i(k+I_c-I_s,t)+$$

$$+\sum_{j=1,j\neq i}^{n}\left[\mu_j(k_j)u(k_j)p_{ji}r_{ij}(k+I_i-I_j,t)-\mu_i(k_i)u(k_i)p_{ij}r_{ji}(k-I_i+I_j,t)\right]+$$

$$+\lambda p_{0i} r_{0i}(k+I_i,t) - \mu_i(k_i)u(k_i)p_{i0}R_{i0}(k-I_i,t)+$$

$$+\left[1-\lambda p_{0i} - \mu_i(k_i)u(k_i) - \sum_{j=1, j\neq i}^{n} \mu_j(k_j)u(k_j)(1-p_{j0})\right] r_i(k), \quad i=\overline{1,n}.$$

Finding a solution of such sets of DDE in general case is rather intricate problem. Different methods of its solution for networks with central QS are described in papers [3 – 5].

Let's consider a qualitative situation with changing of incomes when time t is large. When system's service rate doesn't depend on number of messages in QS, total expected income of network's systems satisfies a set of DDE

$$\frac{d\Theta(k,t)}{dt} = \sum_{i=1}^{n} r_i(k) - \sum_{i=1}^{n} \mu_i u(k_i)\Theta(k,t)+$$

$$+\sum_{i=1}^{n-1} [\mu_i u(k_i)\Theta(k-I_i+I_n,t) + \mu_n u(k_n)p_{ni}\Theta(k+I_i-I_n,t)]. \tag{1}$$

Theorem 1. *For total summary expected income of network $\Theta(t)$ an expression $\Theta(t) = = gt + V + \alpha(t)$ takes place, where $\alpha(t) \underset{t\to\infty}{\to} J_0$, J_0 – zero column vector, g and V are some vectors. Similar relations take place for systems expected incomes if $r(k,t) = = r_0(k)$, $R(k,t) = R(k)$.*

3 Example

It's obvious that stationary solution of set of equations (1) for total incomes of networks systems doesn't exist, when $g = SQ \neq J_0$. However, if $Q = J_0$ then such solution exists and equals V. It is possible to achive that for central QS a condition $r_n(k)+ + \sum_{i=1}^{n-1} [\mu_i u(k_i) r(k-I_i+I_n,t) - \mu_n u(k_n)p_{ni}R(k+I_i-I_n,t)] = 0$ should take place for that equality, and a condition $[\mu_n u(k_n)p_{ni}R(k+I_i-I_n,t) - \mu_i u(k_i)r(k-I_i+I_n,t)] + +r_i(k) = 0$ should take place for incomes of peripheral systems. For example, for the QN with following parameters $n=3$, $K=3$, $\mu_1=1$, $\mu_2=2$, $\mu_3=3$, $p_{31}=0.5$, $p_{32}=0.5$; $r_1(1)=1$, $r_1(2)=2$, $r_1(3)=5$, $r_1(4)=7$, $r_1(5)=9$, $r_1(6)=2$, $r_1(7)=4$, $r_1(8)=6$, $r_1(9)=8$, $r_1(10)=0$; $r_3(1)=5$, $r_3(2)=6$, $r_3(3)=4$, $r_3(4)=0$, $r_3(5)=15$, $r_3(6)=7$, $r_3(7)=0$, $r_3(8)=0$, $r_3(9)=0$, $r_3(10)=0$; $r(1,t)=2$, $r(2,t)=1$, $r(3,t)=0$, $r(5,t)=\frac{3}{2}\sin(t)$, $r(6,t)=0$, $r(8,t)=0$; $R(2,t)=10-\frac{4}{3}\sin(t)$, $R(3,t)=28-\frac{4}{3}\cos(t)$, $R(4,t)=\frac{2}{9}(-48-64\sin(t))$, $R(5,t)=\sin(t)$, $R(6,t)=\cos(t)$, $R(7,t)=20+\frac{32}{3}\sin(t)$, $R(8,t)=\frac{3}{4}(42-\cos(t))$, $R(9,t)=\sin(t)$, $R(10,t)=6\sin(t)$, stationary solution exists for central system's S_3 expected incomes, figure 1, but doesn't exist for peripheral QS S_1, figure 2.

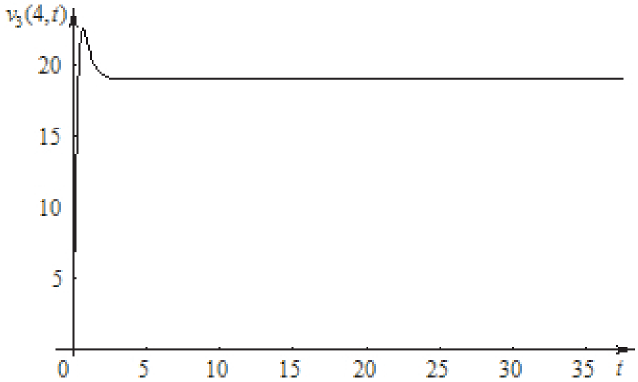

Figure 1: Solution of set (1) for central QS's expected incomes

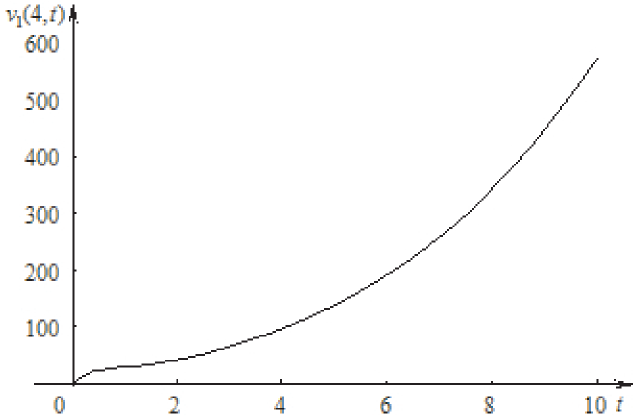

Figure 2: Graph of function $v_1(4, t)$

References

[1] Howard R. (1960). *Dynamic Programming and Markov Processes*. John Willey and Sons, New York.

[2] Sokolov G.A., Chistiakova N.A. (2005). *Probability theory. Controlled Markov chains in economics*. Fizmatlit, M. 246 p.

[3] Matalytski M., Pankov A. (2003). Analysis of stochastic model of the changing of incomes in the open banking network. *Computer Science*. Vol. **3**, N 5. pp. 19-30.

[4] Matalytski M., Pankov A. (2004). The method of approximation for calculation of incomes in banking networks. *Computer Data Analysis and Modelling. Proceedings of 7 Int. Conf.* Minsk: BSU, 2004. Vol. **2**. pp. 88-91.

[5] Matalytski M., Pankov A. (2006). About forecasting of incomes in closed Markov network with central system. *Obozrenie prikladnoy i promyshlennoy matematiki*. Vol. **13**, P. **1**, pp. 123-125.