THE INCREMENT RATIO TEST FOR LONG MEMORY

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Abstract

We introduce a new statistic written as a sum of certain ratios of second order increments of partial sums process $S_n = \sum_{t=1}^{n} X_t$ of observations, which we call the Increment Ratio (IR) statistic. The IR statistic can be used for testing nonparametric hypotheses for $d-$integrated $(-1/2 < d < 3/2)$ behavior of time series X_t, including short memory $(d = 0)$, (stationary) long–memory $(0 < d < 1/2)$ and unit roots $(d = 1)$. If S_n behaves asymptotically as an (integrated) fractional Brownian motion with parameter $H = d + 1/2$, the IR statistic converges to a monotone function $\Lambda(d)$ of $d \in (-1/2, 3/2)$ as both the sample size N and the window parameter m increase so that $N/m \to \infty$. For Gaussian observations X_t, we obtain a rate of decay of the bias $\mathrm{E}IR - \Lambda(d)$ and a central limit theorem $(N/m)^{1/2}(IR - \mathrm{E}IR) \to \mathcal{N}(0, \sigma^2(d))$, in the region $-1/2 < d < 5/4$. Graphs of the functions $\Lambda(d)$ and $\sigma(d)$ are included. A simulation study shows that the IR test for short memory $(d = 0)$ against stationary long–memory alternatives $(0 < d < 1/2)$ has good size and power properties and is robust against changes in mean, slowly varying trends and nonstationarities. We apply this statistic to sequences of squares of returns on financial assets and obtain a nuanced picture of the presence of long–memory in asset price volatility.

References

[1] Berkes I., Horváth L., Kokoszka P.S. and Shao Q-M. (2006) On discriminating between long–range dependence and changes in mean. *Ann. Statist.* Vol. **34**, pp. 1140–1165.

[2] Giraitis L., Kokoszka P., Leipus R. and Teyssière G. (2003). Rescaled variance and related tests for long memory in volatility and levels. *J. Econometrics*. Vol. **112**, pp. 265–294.

[3] Giraitis L., Leipus R. and Philippe A. (2006). A test for stationarity versus trends and unit roots for a wide class of dependent errors. *Econometric Th.* Vol. **22**, pp. 989–1029.

[4] Kwiatkowski D., Phillips P.C.B., Schmidt P. and Shin Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: how sure are we that economic time series have a unit root? *J. Econometrics*. Vol. **54**, pp. 159–178.

[5] Lo A.W. (1991). Long-term memory in stock market prices. *J. Econometrics*. Vol. **59**, pp. 1279–1313.

[6] Lobato I. and Robinson P.M. (1998). A nonparametric test for I(0). *Rev. Econ. Studies*. Vol. **65**, pp. 475–495.

[7] Robinson P.M. (1994). Efficient tests of nonstationary hypotheses. *J. Amer. Statist. Assoc*. Vol. **89**, pp. 1420–1437.

[8] Surgailis D., Teyssière G. and Vaičiulis, M (2007). The increment ratio statistic. *J. Multivariate Analysis Appl.* (to appear).