CLASSIFICATION OF AUTOREGRESSIVE TIME SERIES UNDER "OUTLIERS"

I.K. Pirshtuk
National Research Center for Applied Problems of Mathematics and Informatics
Belarusian State University
Minsk, BELARUS
e-mail: pirshtuk@bsu.by

Abstract

The problem of robust classification of autoregressive time series under "outliers" considered and the asymptotic expansion of the risk of classification is constructed in the paper.

Let random time series $X = (x_1, \dots, x_n) \in \mathbf{R}^n$ of length n belonging to one of $L(L \geq 2)$ classes $\Omega_1, \dots, \Omega_L$ with a priori probabilities $\pi_1, \dots, \pi_L(\pi_1 + \dots + \pi_L = 1)$ is observed. A random time series from the class Ω_i is described by the model of autoregression AR(p) with observations errors:

$$y_t + \theta_{i1} y_{t-1} + \dots + \theta_{ip} y_{t-p} = v_t, \tag{1}$$

$$x_t = y_t + (1 + (K_i - 1)\eta_t^{(i)})\sigma\xi_t, \quad (i = \overline{1, L};\ t = \overline{1, n}), \tag{2}$$

where p is the order of autoregression; $\theta_i = (\theta_{i1}, \dots, \theta_{ip})' \in R^p$ — is the vector of autoregression coefficients (class i); v_t — is sequence of i.i.d. Gaussian random variables with zero mean and the variance $B^2 < \infty$; $\{\xi_t\}$ — is a sequence of i.i.d. random with p.d.f. $f(x)$; $\{\eta_t^{(i)}\}$ — is a sequence of i.i.d. Bernouli random variables:

$$\mathbf{P}\{\eta_t^{(i)} = 1\} = \varepsilon_i, \quad \mathbf{P}\{\eta_t^{(i)} = 0\} = 1 - \varepsilon_i, \quad 0 \leq \varepsilon_i \leq \varepsilon_+. \tag{3}$$

Here $\sigma > 0, K_i >> 1, \varepsilon_i, \varepsilon_+$ — are parameters of the model.

Ignoring of the "outliers" in model (1)—(3) leads to the fact, that the Bayesian decision rule $d = d_o(X) : R^n \to S = \{1, 2, \dots, L\}$, wich minimizes risk at ε_+ (absence distortions), looses its optimality under "outliers" presence $\varepsilon_+ > 0$.

In the paper the following topical problems are solved: robustness evaluation for traditional Bayesian decision rules under "outliers"; estimaation of critical distortions level ε_+, K_i; constuction robust decision rules $d = d_*(X)$.

Computer results are presented for Gaussian model of distortions: $f(z) = n_1(z/0, 1)$.