ALGORITHMS AND DATA STRUCTURES FOR CONSTRUCTION OF THE VECTOR OF VARIATION OF CO-FLOW FOR A DUAL GENERALIZED NON-HOMOGENEOUS NETWORK FLOW PROGRAMMING PROBLEM

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Abstract: In this paper we introduce the algorithm for construction of a vector of variation of co-flow which is used in a dual method for solving non-homogenious generalized network flow programming problem. The algorithm is based on decomposition of the system and the network characteristics of the support. We provide information about data structures which can be used for the algorithm implementation and allow having linear computational complexity for the algorithm.

Key Words: decomposition, network flow programming, sparse matrices, support.

Let us consider the dual non-homogenious network flow programming problems with an interconnection of arc flows [2]. Let $S=\{I,U\}$ be a finite oriented connected network, where I is the set of nodes and U is the set of arcs, $U\subseteq I\times I, (|I|<\infty, |U|<\infty)$. We also introduce a connected network $S^k=\{I^k,\widetilde{U}^k\}$, $I^k\subset I, \widetilde{U}^k\subset U$, each corresponding to some flow type (product) $k\in K, |K|<\infty$. Let $K(i)=\{k\in K: i\in I^k\}$, $i\in I$, $K(i,j)= =\{k\in K:(i,j)^k\in U^k\}$, $(i,j)\in U$. The elements of every network have the following characteristics: $d_{ij}=(d_{ij}^k, k\in K_1(i,j))$ is the vector of capasities of the arc $(i,j)\in U$, $K_1(i,j) =\{k\in K(i,j):(i,j)\in \widetilde{U}_1^{\,k}\}$; d_{ij}^0 is the overall capasity of the arc $(i,j)\in U_0, U_0\subset U$, U_0- a given set, $\sum_{k\in K_0(i,j)} x_{ij}^k\le d_{ij}^0$, $(i,j)\in U_0$ $K_0(i,j)=K(i,j)\setminus K_1(i,j)$, $|K_0(i,j)|>1$.

Let us determine increment of the potentials $\Delta u^k=(\Delta u_i^k, i\in I^k;\quad \Delta r_p, p=\overline{1,q}, \Delta\xi_{ij}, (i,j)\in U^*)$, $k\in K$, where q is a number of additional constraints which provide an interconnection of arc flows [2]. Similar to [2] we introduce the support for generalized multi network.

Consider the following linear system:

$$\Delta\delta_{ij}^k=\Delta u_i^k-\mu_{ij}^k\Delta u_j^k+\sum_{p=1}^{l}\lambda_{ij}^{kp}\Delta r_p=0, \qquad (1)$$

$$(i,j)^k\in U_S^k\setminus U_k^*, k\in K\setminus K_0,$$

$$(i,j)^k\in U_S^k\setminus((i_0,j_0)^k\ \bigcup U_k^*), k\in K_0;$$

$$\Delta\delta_{ij}^k=\Delta u_{i_0}^k-\mu_{ij}^k\Delta u_{j_0}^k+\sum_{p=1}^{l}\lambda_{i_0j_0}^{kp}\Delta r_p=$$

$$=\begin{cases} sign\,\alpha, \text{если } (i_0,j_0)\notin U_k^*; \\ -\Delta\xi_{ij}+sign\,\alpha, \text{если } (i_0,j_0)\in U_k^*, k\in K_o; \end{cases}$$

$$\Delta\delta_{ij}^k=\Delta u_i^k-\mu_{ij}^k\Delta u_j^k+\sum_{p=1}^{l}\lambda_{ij}^{kp}\Delta r_p=-\Delta\xi_{ij},$$

$$(i,j)\in U^*\setminus(i_0,j_0).$$

where $(i_0,j_0)^k$ - an arc not satisfying the optimum criterion [2]; μ_{ij}^k - coefficients of arc flow transformation. Let us introduce the algorithm for solving the system (1) based on decomposition of variables. Let us compute the vector $\varphi=(\Delta r_p, p=\overline{1,l}; \Delta\xi_{ij}, (i,j)\in U^*)$, components of which satisfy the next system:

$$D'\varphi=\beta\, sign(\alpha). \qquad (2)$$

The right-hand side $\beta=(\beta_t, t=\overline{1,|U_B|})$ has the form:

$$\beta_t=\begin{cases} \delta_{i_0j_0}^k(\tau,\rho), if\ (i_0,j_0)^k\in L_t^k \\ \quad and\, arc\, (i_0,j_0)^k-a\, direct\, arc; \\ -\delta_{i_0j_0}^k, if\ (i_0,j_0)^k\in L_t \\ \quad and\, arc\, (i_0,j_0)^k-a\, reverse\, arc; \\ 0, if\ (i_0,j_0)^k\notin L_t^k, \end{cases} \qquad (3)$$

Let us define matrix $D=(\Lambda_{\tau\rho}^{kp}, p=\overline{1,q}, t(\tau,\rho)^k=\overline{1,|U_B|})$, where matrix D has the size

$q \times |U_B|$ and consists of determinants $\Lambda_{\tau\rho}^{kp} = \sum_{(i,j)^k \in L_{\tau\rho}^k} \lambda_{ij}^{kp} \delta_{ij}^k(\tau,\rho)$, where $(\tau,\rho)^k \in U^k \setminus U_L^k$, U_L^k – the support of the network $S^k = (I^k, U^k)$, $k \in K$ for systems of the special form [1].

In case of non-singularity of the matrix D, we find the single-valued unknown variables of the vector $\varphi = (\Delta r_p, p = \overline{1,l};\ \Delta\xi_{ij}, (i,j) \in U^*)$:

$$\varphi = (\Delta r_p, p = \overline{1,l}; \Delta\xi_{ij}, (i,j) \in U^*):$$

$$\varphi = (D')^{-1} \beta\, sign(\alpha). \tag{4}$$

We obtain the system of n equations with n unknowns concerning to the components of variation vector of the potentials Δu^k for every fixed $k \in K$. We solve the system (1) in consideration of its sparseness using the network characteristics of the support [3]. At first, we compute unknown components of the vector Δu^k for every fixed $k \in K$ for the cyclic arcs. Next, we move, using basic operations for collections of root trees [4], from the vertexes of the cycle to hanging vertexes that correspond to support. For fixed $k \in K$ we find the solution for each system with the computational complexity for worst-case – $O(n)$, where $n = |I^k|$ – the number of vertexes of the network $S^k = (I^k, U^k)$.

Let us introduce data structures that we use for solving the system (1):

- t[i] – reverse list of vertexes of dynastic round [4];
- p[i] – the list of the parents for each vertex;
- d[i] – the list of directions for arcs that are included to the support U_L^k;
- g[i] – the list of vertexes of the cyclic arcs according to fixed direction starting from the arbitrary vertex of the cycle.

At first, we build the solution for the unknowns corresponding to the vertexes of the cycle. Consider that we have already computed the components of the vector φ.

Algorithm for solving the system (1) for unknown components of the vector Δu^k, for fixed $k \in K$, corresponding to the vertexes of the cycle:

1) Put $\widetilde{x}_0 = 0$ for the unknown $\Delta u_{g[1]}^k$;
2) Starting from the vertex g[1] and moving to the last vertex of the list g[i], find the value of the unknown $\Delta u_{g[i]}^k$ from the equation corresponding to the arc (g[i], g[i+1]) if the direction of the arcs (g[i], g[i+1]) and the fixed direction of the cycle round are equal, and the arc (g[i+1], g[i]) if the directions are unequal. At the last step, we build the new value x_0 for the unknown $\Delta u_{g[1]}^k$;
3) Calculate the discrepancy ψ_0 according to the formula $\psi_0 = x_0 - \widetilde{x}_0$;
4) Put $\widetilde{x}_1 = 1$. Moving according to the rules describing in 2), find the new value x_1 for the unknown $\Delta u_{g[1]}^k$. According to 3) find ψ_1;
5) Compute $x_1 = \dfrac{\psi_0}{\psi_0 - \psi_1}$ which is the real value of the component $\Delta u_{g[1]}^k$;
6) According to the rules describing in 2), find the real values for components of the vector Δu^k corresponding to the vertexes of the cycle.

Thus, we build the values for components of the vector Δu corresponding to the vertexes of the cycle with the computational complexity $O(3n)$ where n – the number of vertexes of the cycle. Now we substitute the found components to the system (1). Remaining unknowns of the vector Δu^k, for fixed $k \in K$, concerning to the vertexes for the arcs of the support U_L^k that belong to collections of root trees we find according to the following algorithm.

Starting from the every vertex g[i] of the cycle and moving according to the list t[i], find the real values $\Delta u_{t(i)}^k$. Continue until we solve all equations for the components corresponding to root tree arcs.

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