# A BMAP/SM/1 QUEUEING SYSTEM WITH MMAP-UNPUT OF DISASTERS AND TWO OPERATION MODES

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A single-server queueing system with a batch Markovian arrival process (BMAP) and MMAP-input of disasters and two operation modes is considered. Arrival of disaster causes all customers to leave the system instantaneously. The embedded and arbitrary time stationary queue length distributions and performance characteristics are obtained.

*Keywords:* BMAP/SM/1 queue; marked Markovian arrival process of disasters; operation mode; threshold strategy.

## 1. INTRODUCTION

The queueing systems with negative arrivals describe quite adequately the operation of communication networks with loss of information units. For such systems dynamical control of customers arriving can be mean of reduction the expense of information loss without influence on the reason of loss. The queues with controllable mode of operation receive a significant attention in literature [4, 5].

The loss of the information units can be described by a negative arrival which removes a customer from the system or a disaster arrival as a special case of a negative arrival which causes all the customers to leave the system. The theory of negative arrivals has been originated and developed by Gelenbe [6]. The detailed survey of the queues and networks with negative arrivals and disasters is given in [1, 3].

## 2. MODEL

Let us consider a single-server queue with unlimited waiting space having two modes of operation and additional input of disasters.

The $r$-th mode of operation is described as follows. The input into the system is a BMAP (Batch Markovian Arrival Process). This input is controlled by a stochastic process $\nu_t, t \geq 0$ with the state space of $\nu_t$ is $\{0, 1, ..., W\}$. The transitions of process $\nu_t, t \geq 0$ and arrivals of customers in the $r$-th operation mode are performed according to a matrix generating function $D^{(r)}(z) = \sum_{k=0}^{\infty} D_k^{(r)} z^k, |z| \leq 1,\ r = \overline{1,2}$. More detail description of the BMAP can be found in [8].

We assume that service process is of SM-type. It means that successful service times are the sojourn times of a semi-Markovian process $m_t, t \geq 0$. This process has a state space

$\{1, ..., M\}$ and a semi-Markovian kernel $B^{(r)}(x) = \left(B^{(r)}_{m,m'}(x)\right)_{m,m'=\overline{1,M}}$ when the system operates in the $r$-th mode, $r = \overline{1,2}$. The function $B^{(r)}_{m,m'}(x)$ is the conditional distribution function of the sojourn time of the process $m_t, t \geq 0$ in a state $m$ under the condition that the next state is $m'$, $m, m' = \overline{1,M}$. We use the same assumptions about the kernel $B^{(r)}(x)$ as in [9].

The system under consideration has an additional input of of $K$ types of disasters, $R \geq 1$. The arrival of disaster of type $k$ to the busy system interrupts the service and immediately removes all customers from the system. Then the server is recovered during a period having distribution function $G_k(t)$, $k = \overline{1,R}$. If disaster arrives to the empty system or during a recovery period it's ignored by the system. We consider two cases of customers admission during a recovery period:

a) the customers arriving during a recovery period are accumulated;
b) the customers arriving during a recovery period are lost.

The input of disasters is MMAP (Marked Markovian Arrival Process) [7]. This input is directed by a stochastic process $\eta_t, t \geq 0$ having a state space $\{0, 1, ..., N\}$. When the system operates in the $r$-th mode, the transitions of the process $\eta_t, t \geq 0$ are governed by the matrix generating function $F^{(r)}(z) = \sum_{k=0}^{R} F^{(r)}_k z^k, |z| \leq 1$. Transitions of the process $\eta_t$ without generating of disaster are governed by the matrix $F^{(r)}_0$. Transitions of the chain $\eta_t$, which cause the appearance of disaster of type $K$, are governed by the matrix $F^{(r)}_k$, $k = \overline{1,R}$, $r = \overline{1,2}$.

The quality of the system operation is evaluated by the following cost criterion:

$$C = a\Lambda L + c_1 Y^{(1)} + c_2 Y^{(2)} + dV, \tag{1}$$

where $L$ is the average queue length at service completion epoch; $\Lambda^{-1}$ is the average time between service completion epochs; $Y^{(r)}$ is the average fraction of time, when the $r$-th mode is used, $r = \overline{1,2}$; V is the average number of customers lost per time unit; $a, c_1, c_2$ and $d$ are the corresponding cost coefficients. We assume that $a > 0$, $0 < c_1 \leq c_2$, $d \geq 0$.

The operation mode can be changed at a service completion epoch correspondingly to the threshold strategy. Threshold strategy is determined as follows. An integer-valued threshold $j$ is fixed, $j \geq 0$. If a queue length at a given service completion epoch does not exceed $j$, the first mode is selected for the next customer service. Otherwise, the second mode will be used.

In this paper we present the algorithm for calculation of the cost criterion value under the fixed threshold. Having available this algorithm we can find the optimal threshold strategy that minimizes the cost criterion value in some finite set of the threshold values.

## 3. EMBEDDED MARKOV CHAIN

Let $t_n$ be the $n$-th epoch of customer departures from the system. It's a service completion epoch or a disaster arrival epoch at a busy period.

Consider the five-dimensional Markov chain $\xi_n = \{i_n, u_n, \nu_n, \eta_n, m_n\}$, $n \geq 1$, where

- $u_n = 0$, if $t_n$ is an epoch of successful service completion epoch, in this case $i_n$ is the number of customers in the system at an epoch $t_n + 0$, $i_n \geq 0$;
- $u_n = k$, if $t_n$ is an epoch of arrival of disaster of the type $k$, in this case $i_n$ is the number of customers which leave the system at an epoch $t_n$, $k = \overline{1, R}$, $i_n \geq 1$,

$\nu_n$ is the state of arrival directing process $\nu_t$ at the epoch $t_n$, $\nu_n = \overline{0, L}$; $\eta_n$ is the state of disasters directing process $\eta_n$ at the epoch $t_n + 0, \eta_n = \overline{0, N}$ and $m_n$ is the state of service directing process $m_t$ at the epoch $t_n + 0, m_n = \overline{1, M}$.

Introduce into consideration the stationary state probabilities

$$p(i, \nu, \eta, m) = \lim_{n\to\infty} P\{i_n = i, u_n = 0, \nu_n = \nu, \eta_n = \eta, m_n = m\},\ i \geq 0,$$
$$k^{(r)}(i, \nu, \eta, m) = \lim_{n\to\infty} P\{i_n = i, u_n = r, \nu_n = \nu, \eta_n = \eta, m_n = m\}, i \geq 1,$$
$$\nu, \nu' = \overline{0, W}, \eta, \eta' = \overline{0, N}, m, m' = \overline{1, M}, r = \overline{1, R}.$$

Corresponding to the lexicographic order, introduce the vectors $\vec{p}(i, \nu, \eta) = (p(i, \nu, \eta, 1), \cdots, p(i, \nu, \eta, M))$, $\vec{k}^{(r)}(i, \nu, \eta) = (k^{(r)}(i, \nu, \eta, 1), \cdots, k^{(r)}(i, \nu, \eta, M))$, $\vec{p}(i, \nu) = (\vec{p}(i, \nu, 0), \cdots, \vec{p}(i, \nu, N))$, $\vec{k}^{(r)}(i, \nu) = (\vec{k}^{(r)}(i, \nu, 0), \cdots, \vec{k}^{(r)}(i, \nu, N))$, $\vec{p}_i = (\vec{p}(i, 0), \cdots, \vec{p}(i, W))$, $\vec{k}_i^{(r)} = (\vec{k}^{(r)}(i, 0), \cdots, \vec{k}^{(r)}(i, W))$ and generating functions

$$\vec{P}_1(z) = \sum_{i=0}^{j} \vec{p}_i z^i, \quad \vec{P}_2(z) = \sum_{i=j+1}^{\infty} \vec{p}_i z^i, \quad \vec{K}^{(r)}(z) = \sum_{i=1}^{\infty} \vec{k}_i^{(r)} z^i, \quad r = \overline{1, R},\ |z| \leq 1.$$

**Theorem.** *The vector generating functions $\vec{P}_1(z)$, $\vec{P}_2(z)$ and $\vec{K}^{(r)}(z)$ satisfy the following matrix functional equations:*

$$\vec{P}_1(z)(zI - \beta_1(z)) + \vec{P}_2(z)(zI - \beta_2(z)) = \vec{\pi}_0(\Psi(z) - I)\beta_1(z) + \sum_{r=1}^{R} \vec{K}^{(r)}(1) H^{(r)}(z)\beta_1(z),$$

$$\vec{K}^{(r)}(z) = \left[\vec{\pi}_0(\Psi(z) - I) + \vec{P}_1(z) + \sum_{u=1}^{R} \vec{K}^{(u)}(1) H^{(u)}(z)\right] S_1^{(r)}(z) + \vec{P}_2(z) S_2^{(r)}(z), \quad r = \overline{1, R},$$

*where* $\vec{\pi}_0 = \vec{p}_0 + \sum_{r=1}^{R} \vec{K}^{(r)}(1) H_0^{(r)}$, $\beta_r(z) = \sum\limits_{l=0}^{\infty} \Omega_l^{(r)} z^l = \int\limits_0^{\infty} e^{D^{(r)}(z)t} \otimes e^{F_0^{(r)}t} \otimes dB^{(r)}(t)$, $r = \overline{1, 2}$,

$$\Psi(z) = \sum_{k=1}^{\infty} \Psi_k z^k = -[(D_0^{(1)} \oplus F^{(1)}(1))^{-1}((D^{(1)}(z) - D_0^{(1)}) \otimes I_{N+1})] \otimes I_M,$$

$$S_r^{(u)}(z) = \sum_{k=0}^{\infty} S_{r,k}^{(u)} z^k = \int_0^{\infty} e^{D^{(r)}(z)t} \otimes (e^{F_0^{(r)}t} F_u^{(r)}) \otimes (B^{(r)}(+\infty) - B^{(r)}(t))\, dt, r = \overline{1, 2}, u = \overline{1, R},$$

$$H^{(r)}(z) = \sum_{k=0}^{\infty} H_k^{(r)} z^k = \int_0^{\infty} e^{D^{(1)}(z)t} \otimes e^{F^{(1)}(1)t} dG_r(t) \otimes I_M$$

*– if the customers are accumulated during a recovery period,*

$$H^{(r)}(z) = \sum_{k=0}^{\infty} H_k^{(r)} z^k = H_0^{(r)} = \int_0^{\infty} e^{D^{(1)}(1)t} \otimes e^{F^{(1)}(1)t} dG_r(t) \otimes I_M$$

*– if the customers are lost during a recovery period. Here ⊗ and ⊕ are the symbols of the Kronecker product and Kronecker sum, $I_n$ denotes an identity matrix of the size n.*

**Corollary.** *The generating function* $\vec{\Pi}_1(z)$ *is determined by the equation* $\vec{P}_1(z) = \vec{\pi}_0 Y(z) + \sum_{r=1}^{R} \vec{K}^{(r)}(1) Q_r(z)$, *where* $Y(z) = \sum_{i=0}^{J} Y_i z^i$, $Q^{(r)}(z) = \sum_{i=0}^{J} Q_i^{(r)} z^i$, $r = \overline{1,R}$, *and matrices* $Y_i$ *and* $Q_i$, $i = \overline{0,j}$ *are calculated from the recurrent formulas* $Y_0 = I$, $Y_1 = (\Omega_0^{(1)})^{-1} - \Psi_1$, $Q_1^{(r)} = -H_0^{(r)}(\Omega_0^{(1)})^{-1} - H_1^{(r)}$, $Y_{i+1} = \left(Y_i - \sum_{k=1}^{i+1} \Psi_k \Omega_{i-k+1}^{(1)} - \sum_{k=1}^{i} Y_k \Omega_{i-k+1}^{(1)}\right)(\Omega_0^{(1)})^{-1}$, $Q_0^{(r)} = -H_0^{(r)}$, $Q_{i+1}^{(r)} = (Q_i^{(r)} - \sum_{k=1}^{i} (Q_k^{(r)} + H_k^{(r)})\Omega_{i-k+1}^{(1)})(\Omega_0^{(1)})^{-1} - H_{i+1}^{(r)}$, $i = \overline{1, j-1}$, $r = \overline{1,R}$.

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