SOME NETWORK MODELS WITH NEGATIVE CUSTOMERS AND OTHER SIGNALS

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We will consider the equilibrium product form distribution results for some models of queueing networks with negative customers and other information signals in this paper. This results where obtained by mathematicians of Gomel Probability Subschool of Medvedev Belorussian Probabilistic School. Considered networks are some extensions of basic Gelenbe networks [1, 2] and some another networks.

1. THE NETWORKS WITH BYPASSES OF NODES BY THE CUSTOMERS

The next network was considered in [3]. The customers arrive in network with N exponential single line nodes with service rates μ_i $(i = \overline{1,N})$ according two independent Poisson processes. The first process customers are positive with arrive rate λ and the second process customers are negative with rate ν. The ich positive customer is directed to node i with probability p_{0i}^+ and the ich negative one is directed to the same node with probability p_{0i}^- $\left(\sum_{i=1}^{N} p_{0i}^+ = \sum_{i=1}^{N} p_{0i}^- = 1\right)$. Let us n_i is the number of positive customers in node i. The positive customer directed to node i addes to node queue with probability $f_i(n_i)$ and bypasses node i with probability $1 - f_i(n_i)$ $(0 \le f_i(n_i) \le 1)$. Positive customer bypassing the node has the same behavior as serviced positive customer which is directed to node j with probability p_{ij}^+ as a positive customer and with probability p_{ij}^- as a negative customer or leaves the network with probability p_{i0} $\left(\sum_{i=1}^{N}(p_{ij}^+ + p_{ij}^-) + p_{i0} = 1\right)$. The routing matrix $(p_{ij}^+,\ i,j = \overline{1,N})$ with $p_{00}^+ = 0$ is irreducible. Let us $n_i(t)$ is the number of positive customers in node i at moment t. Then process $n(t) = (n_1(t), n_2(t), \ldots, n_N(t))$ is irreducible Markov process with continuous time and state space $X = X_1 \times X_2 \times \ldots X_N$. Here $X_i = \{0, 1, \ldots\}$ if all $f_i(n_i) > 0$ $(n_i = 0, 1, \ldots)$, and $X_i = \{0, 1, \ldots k_i\}$ if all $f_i(n_i) > 0$ $(n_i = \overline{1, k_i - 1})$ and for some $k_i \ge 1$ it is $f_i(k_i) = 0$ $(i = \overline{1,N})$. From [2] it follows that traffic equations

$$\lambda_i = \sum_{j=1}^{N} \lambda_j \frac{\mu_j}{\mu_j + \nu_j} p_{ji}^+ + \lambda p_{0i}^+, \qquad \nu_i = \sum_{j=1}^{N} \nu_j \frac{\mu_j}{\mu_j + \nu_j} p_{ji}^- + \nu p_{0i}^-, \quad i = \overline{1,N},$$

have a positive solution. The product form equilibrium distribution for this process doesn't exist. Some modification of our model in which we admit fictitious transitions from state

n_l to the same state n_l for isolated node with some certain rate reduce to a new Markov process. This process can be countered as some approximation of the origin process. So we w'll be used the same notation $n(t)$ for this process that for origin process. The result of [3] is

Theorem 1. *If conditions*

$$\sum_{n_i \in X_i} \prod_{k=1}^{n_i} \frac{\lambda_i f_i(k-1)}{\mu_i + \nu_i} < \infty \quad (i = \overline{1,N})$$

are hold then approximative Markov process $n(t)$ is ergodic and its stationary distribution has the next form:

$$p(n) = p_1(n)p_2(n)\dots p_N(n_N), \quad n \in X \tag{1}$$

where

$$p_i(n_i) = p_i(0) \prod_{k=1}^{n_i} \frac{\lambda_i f_i(k-1)}{\mu_i + \nu_i}, \quad i = \overline{1,N}, \quad n_i \in X_i.$$

The additional assumption was introduced in [4] for this model: the waiting time of customers in node l is bounded by some random variable which has exponential distribution with mean θ_l^{-1}. The result is the next extension.

Theorem 2. *If conditions*

$$\sum_{n_i \in X_i} \prod_{k=1}^{n_i} \frac{\lambda_i f_i(k-1)}{\mu_i + \nu_i + (k-1)\theta_l} < \infty \quad (i = \overline{1,N})$$

are hold then approximative Markov process $n(t)$ is ergodic and its stationary distribution has form (1) where

$$p_i(n_i) = p_i(0) \prod_{k=1}^{n_i} \frac{\lambda_i f_i(k-1)}{\mu_i + \nu_i + (k-1)\theta_l}, \quad i = \overline{1,N}, \quad n_i \in X_i.$$

This result was extended on the networks with multiserver nodes in [5].

2. THE NETWORKS WITH MULTIREGIME SERVICE STRATEGIES

The next network consisting of N single line queues was considered in [6,7]. Two independent Poisson flows arrive: positive customers with rate λ^+ and negative customers with rate λ^-. Any positive customer (negative customer) is directed in node l with probability π_{0l}^+ (π_{0l}^-) $(l = \overline{1,N}; \sum_{l=1}^N \pi_{0l^+} = \sum_{l=1}^N \pi_{0l^-} = 1)$. After completing the service in node l positive customer is directed in node m as positive customer with probability π_{lm}^+, as negative customer with probability π_{lm}^-, or leaves the network with probability π_{l0} $(l, m = \overline{1,N}; \sum_{m=1}^N (\pi_{lm}^+ + \pi_{lm}^-) + \pi_{l0} = 1)$. Single server in node l acts on $r_l + 1$ regimes. The state of node l is $x_l = (i_l, j_l)$ where i_l is number of positive customers in node l and j_l is regime order number of server's action in this node $(l = \overline{1,N}; j_l = \overline{0,r_l})$. Service time in node l has the exponential distribution with rate $\mu(l)$. Time epoch in regime 0 (basic regime) has exponential distribution with parameter $\nu_{i_l,0}(l)$ and after its completion the server regime in node l transforms on regime 1. For states x_l with $1 \le j_l \le r_l - 1$ time epoch in regime j_l also has exponential distribution. After its completion the server regime in node l can change with rate $\varphi_{x_l}(l)$ on regime $j_l - 1$ or with rate $\nu_{x_l}(l)$ on regime $j_l + 1$. The time epoch in regime r_l also has exponential distribution with parameter $\varphi_{i_l,r_l}(l)$. After that the server regime in node l transforms on regime $r_l - 1$. The network state at time t is vector $x(t) = (x_1(t), \dots, x_N(t))$ where $x_l(t) = (i_l(t), j_l(t))$ is state of node l at time t. Assume all quantities $\mu(l), \nu_{x_l}(l), \varphi_{x_l}(l)$ are strong positive. Let us α_l^+ (α_l^-) is expected arrive rate of positive (negative) customers in node l. They satisfy the following nonlinear traffic equations:

$$\alpha_l^+ = \lambda^+ \pi_{0l}^+ + \sum_{k=1}^{N} \frac{\alpha_k^+ \mu(k)}{\mu(k) + \alpha_k^-} \pi_{kl}^+; \qquad \alpha_l^- = \lambda^- \pi_{0l}^- + \sum_{k=1}^{N} \frac{\alpha_k^+ \mu(k)}{\mu(k) + \alpha_k^-} \pi_{kl}^-. \tag{2}$$

From [2] this equations have a positive solution on usual assumptions. So Markov process $x(t)$ is irreducible on state space $X = X_1 \times X_2 \times \ldots \times X_N$ where $X_l = \{(i_l, j_l) : i_l = 0, 1, \dots; j_l = 0, 1, \dots, r_l\}$.

Theorem 3. *If conditions*

$$\nu_{i_l,j_l-1}(l)\varphi_{i_l-1,j_l}(l) = \nu_{i_l-1,j_l-1}(l)\varphi_{i_l,j_l}(l), \qquad i_l \ge 1, \ \ 1 \le j_l \le r_l \tag{3}$$

and

$$\frac{\alpha_l^+}{\mu(l) + \alpha_l^-} < 1, \ \ \sup_{x_l \in X_l} [\nu_{x_l}(l) + \varphi_{x_l}(l)] = c_l < \infty \tag{4}$$

are satisfied for all $l = \overline{1, N}$ nodes then Markov process $x(t)$ is ergodic and its equilibrium distribution has the next product form:

$$p_x = p_{x_1}(1) p_{x_2}(2) \dots p_{x_N}(N) \tag{5}$$

where

$$p_{i_l,j_l}(l) = \Big(\frac{\alpha_l^+}{\mu(l) + \alpha_l^-}\Big)^{i_l} \prod_{k=0}^{j_l-1} \frac{\nu_{0,k}(l)}{\varphi_{0,k+1}(l)} p_{00}(l).$$

The next extension [6, 7] was obtained in [8]. Two additional independent Poisson flows introduced here: signals of regime increase with rate ω^+ and signals of regime decrease with rate ω^-. If the signal of regime increase arrives in node l then it switches regime $j_l < r_l$ on regime $j_l + 1$ and keeps regime r_l. If the signal of regime decrease arrives in node l then it switches regime $j_l \ge 1$ on regime $j_l - 1$ and keeps regime 0. After that the signals disappear and don't exert influence on network. Arriving in network positive customers, negative customers, signals of regime increase and signals of regime decrease are directed in node l with the probabilities $\pi_{0l}^+, \pi_{0l}^-, q_{0l}^+, q_{0l}^-$ $\big(l = \overline{1, N};\ \sum_{l=1}^N \pi_{0l}^+ = \sum_{l=1}^N \pi_{0l}^- = \sum_{l=1}^N q_{0l}^+ = \sum_{l=1}^N q_{0l}^- = 1\big)$ respectively. After completing the service in node l positive customer is directed in node m as positive customer with probability π_{lm}^+, as negative customer with probability π_{lm}^-, as signal of regime increase with probability q^+_{lm}, as signal of regime decrease with probability q^-_{lm}, or leaves the network with probability π_{l0} $\left(\sum_m (\pi^+_{lm} + \pi^-_{lm} + q^+_{lm} + q^-_{lm}) + \pi_{l0} = 1\right)$. Let us β^+_l and β^-_l are expected arrive rates of regime increase and regime decrease signals in node l respectively. They can be obtained as

$$\beta^+_l = \omega^+\sigma^+_{0l} + \sum_k \frac{\alpha^+_k \mu(k)}{\mu(k) + \alpha^-_k} q^+_{kl}; \qquad \beta^-_l = \omega^-\sigma^-_{0l} + \sum_k \frac{\alpha^+_k \mu(k)}{\mu(k) + \alpha^-_k} q^-_{kl},$$

where α^+_k, α^-_k satisfy (2).

Theorem 4. *If conditions (3) and (4) are satisfied for all $l = \overline{1,N}$ nodes then Markov process $x(t)$ is ergodic and its equilibrium distribution has product form (5) where*

$$p_{i_l,j_l}(l) = \left(\frac{\alpha^+_l}{\mu(l) + \alpha^-_l}\right)^{i_l} \prod_{s=1}^{j_l} \frac{\nu_{0,s-1}(l) + \beta^+_l}{\varphi_{0,s}(l) + \beta^-_l} p_{0,0}(l), \qquad (i_l, j_l) \in X_l.$$

The open networks with multiregime service strategies and signals acting some exponential random time epoch are considered in [9,10]. Two independent Poisson flows of positive customers and signals arrive with rates λ^+ and λ^- respectively. Let us π^+_{0l} and π^-_{0l} are the probabilities that they be directed in node l respectively ($l = \overline{1,N}$). Also let k_l is the number of positive customers and n_l is the number of signals in node l.The positive customer has exponential distributed service time with parameter $\mu^+_l(k_l)$ for node l and after it completing one moves in node m as a positive customer with probability π^+_{lm}, as a sygnal with probability π^-_{lm}, or leaves network with probability π_{l0}. Ich signal entering in node l acts during random time which has exponential distribution with rate $\mu^-_l(n_l)$. When this duration is finished a signal moves positive customer from node l to node m with probability σ^+_{lm} as a positive customer, with probability σ^-_{lm} as a signal, or deastroys some positive customer and vanishes with probability $\sigma_{l0} = 1 - \sum_{m=1}^{N}(\sigma^+_{lm} + \sigma^-_{lm})$. The signal disappears when there are not positive customers in node l. The parameters of regimes are determinated so as above and they dependent on (k_l, j_l) where j_l is order number of regime for node l. The network state is vector $x = (x_1, x_2, \ldots, x_N)$ where $x_l = (k_l, n_l, j_l)$ is state of node l.

The case $\mu_l(k_l)^+ = \mu_l$ for $k_l > 0$ was considered in [9]. Let we have a positive solution of traffic equations

$$\alpha^+_l = \lambda^+\pi^+_{0l} + \sum_k \frac{\alpha^+_k(\mu^+_k\pi^+_{kl} + \alpha^-_k\sigma^+_{kl})}{\alpha^-_k + \mu^+_k}; \quad \alpha^-_l = \lambda^-\pi^-_{0l} + \sum_k \frac{\alpha^+_k(\mu^+_k\pi^-_{kl} + \alpha^-_k\sigma^-_{kl})}{\alpha^-_k + \mu^+_k}.$$

Theorem 5. *If conditions*

$$\alpha^+_l < \alpha^-_l + \mu^+_l; \quad \sum_{n_l=0}^{\infty}\prod_{s=1}^{n_l} \frac{\alpha^-_l}{\mu^-_l(s)} < \infty; \quad \sup_{k_l,j_l} \nu_l(k_l, j_l) < \infty;$$

$$\forall k_l, j_l \quad \nu_l(k_l, j_l - 1) < M\varphi_l(k_l, j_l) \quad \textit{for some} \quad M < 1$$

are satisfied for all $l = \overline{1,N}$ nodes then Markov process $x(t)$ is ergodic and its invariant measure has product form

$$p(k, n, j) = \prod_{l=1}^{N} p_l(k_l, n_l, j_l),$$

where

$$p_l(k_l, n_l, j_l) = \Big(\frac{\alpha_l^+}{\alpha_l^- + \mu_l^+}\Big)^{k_l} \prod_{s=1}^{n_l} \frac{\alpha_l^-}{\mu_l^-(s)} \prod_{u=0}^{j_l} \frac{\nu_l(k_l, u-1)}{\varphi_l(k_l, u)}.$$

Restriction $\mu_l(k_l)^+ = \mu_l$ for $k_l > 0$ took off in [10] but there are another limitation

$$\pi_{lm}^+ = \sigma_{lm}^+ = p_{lm}^+, \quad \pi_{lm}^- = \sigma_{lm}^- = p_{lm}^-, \quad \pi_{l0} = \sigma_{l0} = p_{l0},$$

i. e. this network is symmetric.

3. THE NETWORKS WITH GENERAL NODE DESCRIPTION

Next open network with N single server nodes was considered in [11]. Two independent Poisson flows arrive: positive customers with rate λ^+ and negative customers with rate λ^-. Positive (negative) customers are directed in node l with probability p_{0l}^+ (p_{0l}^-), $l = \overline{1,N}$. Servered in node l customer immediately is directed in node m as positive with probability p_{lm}^+, as negative with probability p_{lm}^-, or leaves the network with probability p_{l0}. The network state is $x(t) = (x_1(t), \ldots, x_N(t))$ where $x_l(t)$ is the state of node l. Let $|x_l|$ is the number of positive customers in node l in state x_l. When positive (negative) customer arrives in node l the state of node transforms from x_l to $\tilde{x}_l$ with probability

$$\pi_l^+(x_l, \tilde{x}_l), \quad |\tilde{x}_l| = |x_l| + 1,$$

$$\pi_l^-(x_l, \tilde{x}_l), \quad |\tilde{x}_l| = |x_l| - 1,$$

$$\Big(\sum_{|\tilde{x}_l|=|x_l|+1} \pi_l^+(x_l, \tilde{x}_l) = \sum_{|\tilde{x}_l|=|x_l|-1} \pi_l^-(x_l, \tilde{x}_l) = 1\Big).$$

Service time in node l has exponential distribution with rate μ_l and after it completing the state of node l transforms from x_l to $\tilde{x}_l$ with probability $\rho_l(x_l, \tilde{x}_l)$, $|\tilde{x}_l| = |x_l| - 1$ $\Big(\sum_{|\tilde{x}_l|=|x_l|-1} \rho_l(x_l, \tilde{x}_l) = 1,\ l = \overline{1,N}\Big)$. Let us $(\alpha_1^+, \ldots, \alpha_N^+;\ \alpha_1^-, \ldots, \alpha_N^-)$ is a positive solution of traffic equations

$$\alpha_l^+ = \lambda^+ p_{0l}^+ + \sum_k \frac{\alpha_k^+ \mu_k}{\mu_k + \alpha_k^-} p_{kl}^+; \quad \alpha_l^- = \lambda^- p_{0l}^- + \sum_k \frac{\alpha_k^+ \mu_k}{\mu_k + \alpha_k^-} p_{kl}^-; \quad l = \overline{1,N}.$$

We assume the process of the states for isolated node in fictitious environment is reversible. Its equilibrium distribution

$$p_l(x_l) = p_l(0) \prod_{k=1}^{|x_l|} \frac{\alpha_l^+ \pi_l^+(u_{l,k-1} u_{l,k})}{\mu_l \rho_l(u_{l,k} u_{l,k-1}) + \alpha_l^- \pi_l^-(u_{l,k} u_{l,k-1})}\Big|_{|u_{l,k}|=k} \tag{6}$$

doesn't depend on path form $0 \to u_{l,1} \to u_{l,2} \to \ldots \to u_{l,|x_l|-1} \to x_l$ from state 0 to state x_l.

Theorem 6. *If inequalities* $\alpha_m^+ < \mu_m + \alpha_m^-$ *and conditions*

$$\sum_{|\tilde{x}_m|=|x_m|+1} p_m(\tilde{x}_m) \rho_m(\tilde{x}_m, x_m) = \sum_{|\tilde{x}_m|=|x_m|+1} p_m(\tilde{x}_m) \pi_m^-(\tilde{x}_m, x_m)$$

are satisfied for all $m = \overline{1,N}$ nodes then process $x(t)$ is ergodic and its equilibrium distribution has product form $p(x) = p_1(x_1)\dots p_N(x_N)$ where the factors are determined by (6).

This result is keep if we additionaly suppose that there are internal transitions in nodes with some rate $\nu_l(x_l, \tilde{x}_l)$ from state x_l to state $\tilde{x}_l$, $|x_l| = |\tilde{x}_l|$.

4. INSENSITIVITY OF STATIONARY DISTRIBUTION

The open network consisting of N single server nodes was considered in [12]. Two independent Poisson flows enter network: positive customers with rate λ^+ and negative customers with rate λ^-. Positive (negative) customers are directed in node l with probability

$$p^+_{0l},\quad l = \overline{1,N},\quad \sum_{l=1}^{N} p^+_{0l} = 1 \quad \left(p^-_{0l},\quad l = \overline{M+1,N},\quad \sum_{l=M+1}^{N} p^-_{0l} = 1\right).$$

Arriving in node $l = \overline{1,M}$ positive customer begin to receive the service with probability $f_l(n_l)$ depending on number of customers in node l, or bypasses node l with probability $1 - f_l(n_l)$ and with routing the same as one has customer after its service in node l. Arriving in node $l = \overline{M+1,N}$ positive customer begin to receive the service immediately. Servered in node l $(l = \overline{1,N})$ positive customer immediately is directed in node m as positive with probability p^+_{lm}, as negative with probability p^-_{lm}, or leaves the network with probability p_{l0}. Service discipline in all nodes is LCFS PR here. Single server in node $l = \overline{M+1,N}$ is exponential with the rate μ_l and in node $l = \overline{1,M}$ one has arbitrary general distribution $B_i(n_l, t)$ such that

$$\mu_i^{-1}(n_i) = \int_0^{+\infty} [1 - B_i(n_i, t)]dt, \qquad i = \overline{1,M}.$$

If matrix $P^+ = (p^+_{lm},\ \ l, m = \overline{0,N})$ with $p^+_{00} = 0$ is irreducible then traffic equation

$$\lambda^+_l = \sum_{k=1}^{M} \lambda^+_k p^+_{kl} + \sum_{k=M+1}^{N} \frac{\lambda^+_k \mu_k}{\mu_k + \lambda^-_k} p^+_{kl} + \lambda^+ p^+_{0l}, \qquad l = \overline{1,N},$$

$$\lambda^-_l = \sum_{k=1}^{M} \lambda^+_k p^-_{kl} + \sum_{k=M+1}^{N} \frac{\lambda^+_k \mu_k}{\mu_k + \lambda^-_k} p^-_{kl} + \lambda^- p^-_{0l}, \qquad l = \overline{M+1,N}$$

has positive solution. The network state is $n(t) = (n_1(t), n_2(t) \dots n_N(t))$ where $n_l(t)$ is number of positive customers in node l at moment t. The state space is $\mathbf{X} = X_1 \times X_2 \times \dots \times X_N$ where for $l = \overline{1,M}$ $X_l = \{0, 1, 2, \dots\}$ if all $f_l(n_l) > 0\, (n_l = 0, 1, \dots)$ and $X_l = \{0, 1, \dots, k_l\}$ if all $f_l(n_l) > 0\, (n_l = 0, 1, \dots, k_l - 1)$ and $f_l(k_l) = 0$ for some $k_l \geq 1$; $X_l = \{0, 1, 2, \dots\}$ if $l = \overline{M+1,N}$.

Theorem 7. *If the next conditions*

$$\sum_{n_i \in X_i} \prod_{k=1}^{n_i} \frac{\lambda^+_i f_i(k-1)}{\mu_i(k)} < +\infty, \quad i = \overline{1,M}, \quad \sum_{n_i \in X_i} \prod_{k=1}^{n_i} \frac{\lambda^+_i}{\mu_i + \lambda^-_i} < +\infty, \quad i = \overline{M+1,N}$$

are satisfied then Markov process $n(t)$ is ergodic and its invariant measure has product form $p(x) = p_1(x_1)\dots p_N(x_N)$ where

$$p_i(n_i) = \begin{cases} \prod\limits_{k=1}^{n_i} \frac{\lambda_i^+ f_i(k-1)}{\mu_i(k)} & \text{for } i = \overline{1,M}, \\ \prod\limits_{k=1}^{n_i} \frac{\lambda_i^+}{\mu_i+\lambda_i^-} & \text{for } i = \overline{M+1,N}. \end{cases}$$

It follows from Theorem 7 that steady state distribution doesn't depend on service time distributions in nodes $l = \overline{1,M}$ when these distributions have fixed first moments. Similar result is obtained in [13] when the service discipline in nodes is EPS.

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