ON ASYMPTOTIC ENLARGEMENT PROBLEM FOR STOCHASTIC NETWORKS

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In connection with stochastic networks the problem of phase enlargement is discussed. For models of the $[G|GI|\infty]^r$ type it's given a variant of conditions which result in an enlargement of nodes and diffusion approximation of the process of information treatment.

Keywords: phase enlargement, stochastic network, diffusion and Gaussian approximation.

First for the complicated stochastic system the problem of phase enlargement was considered by V. S. Korolyuk (see [1]). Later on many authors dealt with the problem. Among them we single out the works of V. V. Anisimov [2] and A. F. Turbin [3].

For stochastic networks an asymptotic enlargement takes place in such situations when the connections between selected classes of nodes are "asymptotically small" in comparison with interior connections in the classes. Under passage to the limit the class of nodes behaves as a node and the limit process has a dimension which is equal to the number of classes. As analysis has shown for stochastic networks the problem of phase enlargement has two peculiarities. In the first place the enlargement of network nodes takes place in process of diffusion or Gaussian approximation. Secondly, the enlargement is relative only to the dimension of the process of information treatment.

For multi-channel stochastic network of the type $[G|GI|\infty]^r$ we formulate a variant of conditions which lead to asymptotic enlargement of nodes.

We will assume that the parameters of $[G|GI|\infty]^r$-network depends on "n" (a number of series) and the following holds true:

1) For input flow $\nu^n(t) = (\nu^{(1)n}(t), \ldots, \nu^{(r)n}(t))$ there exist constants $\lambda_i \geq 0$, $i = 1, 2, \ldots, r$, $\lambda_1 + \ldots + \lambda_r \neq 0$ that

$$n^{-1/2}(\nu^{(1)n}(nt) - \lambda_1 nt, \ldots, \nu^{(r)n}(nt) - \lambda_r nt) \overset{U}{\Rightarrow} W'(t) = (W_1(t), \ldots, W_r(t)),$$

where $W(t)$ is r-dimensional process of Brownian motion with a null-vector of mean values and a correlation matrix $MW(1)W'(1) = \sigma^2 = \|\sigma_{ij}\|_1^r$.

2) A set of network nodes $I = \{1, 2, \ldots, r\}$ is decomposed into classes $I_1, \ldots, I_{r_0}$ ($I_i \cap I_j = \phi$, $i \neq j$, $i, j = 1, \ldots, r_0$) in a such way that for switching matrix $P_n = \|p_n(i, j)\|_{i,j \in I}$ the following relations take place

i) $P_n = P_0 + n^{-1}B_0 + o(n^{-1})$, $p_n(i, r+1) = n^{-1}b_{ir+1} + o(n^{-1})$,

$i = 1, 2, \ldots, r$, where $P_0 = \|\delta_{\alpha\beta}P^{(\alpha)}\|_1^{r_0}$, $P^{(\alpha)} = \|p_{ij}^{(\alpha)}\|_{i,j\in I_\alpha}$ is an indecomposable stochastic matrix with the stationary distribution $\rho_i^{(\alpha)}$, $i \in I_\alpha$; $B_0 = \|B_{\alpha\beta}\|_1^{r_0}$, $B_{\alpha\beta} = \|b_{ij}\|_{i\in I_\alpha}$, $j \in I_\beta$ are rectangular matrixes of the dimension $|I_\alpha| \times |I_\beta|$, $\alpha, \beta = 1, 2, \ldots, r_0$;

ii) $a_{\alpha\alpha} = -\sum\limits_{\beta=1,\beta\neq\alpha}^{r_0+1} a_{\alpha\beta} \neq 0$,

where $a_{\alpha\beta} = \rho'^{(\alpha)}B_0 1^{(\beta)}$, $\alpha, \beta = 1, \ldots, r_0$, $\alpha \neq \beta$, $a_{\alpha r_0+1} = \rho'^{(\alpha)}b_{r+1}$, $\alpha = 1, \ldots, r_0$, $b'_{r+1} = (b_{1r+1}, b_{2r+1}, \ldots, b_{rr+1})$, $\rho^{(\alpha)}$ is r-dimensional vector with i-th component $\rho_i^{(\alpha)}$ under $i \in I_\alpha$ and with 0 in the opposite case; $1^{(\beta)}$ is r-dimensional vector with i-th component 1 under $i \in I_\beta$ and with 0 in the opposite case.

iii) The spectral radius of $\hat{P} = \|\hat{p}_{\alpha\beta}\|_1^{r_0} = \|(1 - \delta_{\alpha\beta})a_{\alpha\beta}/(-a_{\alpha\alpha})\|_1^{r_0}$ is less, than 1.

3) Distribution functions $G_i(t)$, $i = 1, 2, \ldots, r$ of packet treatment time in nodes of the network do not depend on parameter "n" and there exist their mean values

$$\int_0^\infty t dG_i(t) = 1/\mu_i < \infty, \quad i = 1, 2, \ldots, r.$$

If conditions 1)–3) hold true and at initial moment of time the $[G|GI|\infty]^r$-network is empty then it is possible to enlarge nodes of the network and to approximate the process of information treatment by the diffusion of dimension r_0 which is equal to the number of classes. A drift and a matrix of the diffusion can be written through the network parameters in explicit form. The principal premise of enlargement is condition 2). Modifications of 1), 3) essentially define the type of approximate process.

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