NUMERICAL SIMULATION OF NONLINEAR EFFECTS IN VOLUME FREE ELECTRON LASER (VFEL)

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Abstract

In this paper a mathematical model of Volume Free Electron Lasers (VFEL) is described. It proved itself be effective in simulation of different schemes of VFEL. Numerical results obtained confirm all originated in VFEL physical laws.

INTRODUCTION

First lasing of Volume Free Electron Lasers (VFEL) in mm wavelength range was obtained recently [1]. So-called volume multi-wave distributed feedback is the distinctive feature of VFEL.

It is well-known, that the FEL lasing can be result of different types of spontaneous emission mechanisms: undulator radiation, Smith-Purcell or Cherenkov radiation and etc. Using positive feedback in FELs reduces the working length and provides oscillation regime of generation. This feedback is usually one-dimensional and can be formed either by two parallel mirrors or by one-dimensional diffraction grating, in which incident and diffracted waves move along the electron beam. Theoretical investigations show that it is one of the effective schemes with n-wave volume distributed feedback (VDFB) where waves and electron beam spread angularly one to other.

The principles and theoretical foundations of VFEL operation based on mechanism of multi-wave VDFB were proposed in [2]. There it was shown that the increment of instability for an electron beam passing through a spatially-periodic target in degeneration points essentially increased in comparison with single-wave system. This means the noticeable reduction of electron beam current density necessary for achievement the generation threshold. In X-ray range this generation threshold can be reached for the induced parametric X-ray radiation in crystals. It enables to create X-ray laser. This valid for all wavelength ranges regardless the spontaneous radiation mechanism. Prototypes of VFEL based on induced radiation in three-dimensional periodical structures were investigated in [2] and [3].

In VFEL operation the linear stage investigated in [2] - [4] quickly changes into the nonlinear one where most of the electron beam energy is transformed into electromagnetic radiation. A detailed numerical analysis of this stage is necessary for experiment design, optimal geometry determination and result processing.

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VFEL SCHEMES

VFEL resonator of the experimental installation [1] is formed by two diffraction gratings with different periods and two smooth side walls. The interaction of the exciting diffraction grating with the electron beam induces Smith-Purcell radiation. The resonant grating provides distributed feedback of generated radiation with electron beam by Bragg dynamical diffraction. Resonator design allows varying its parameters during experiment. Exciting grating can move to change the distance between gratings and the gap between exciting grating and electron beam. Resonant grating can rotate to change orientation of grating grooves with respect to electron beam velocity that provides possibility of tuning of Bragg diffraction conditions.

Volume resonator (so-called ”grid” volume resonator) of the installation [5] is formed by a periodic structure built from the metallic threads inside a rectangular waveguide. Then the effect of anomalous transmission for electromagnetic waves could appear similarly to the Bormann effect well-known in the dynamical diffraction theory of X-rays [5]. The second (resonant) grating provides the distributed feedback of generated radiation with electron beam by Bragg dynamical diffraction.

These both schemes can be reduced to the following simple scheme of VFEL (see Fig.1) by recounting of dielectric susceptibility of the target.

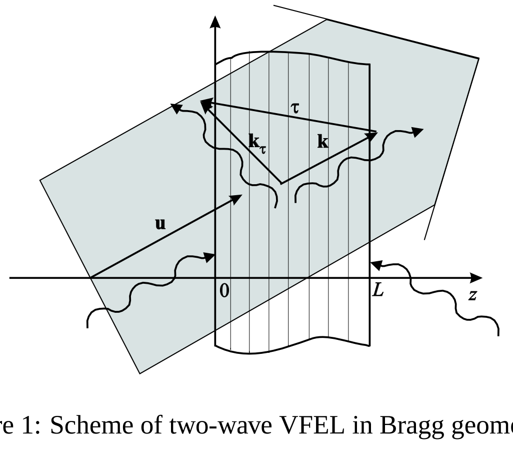

Figure 1: Scheme of two-wave VFEL in Bragg geometry.

Here an electron beam with electron velocity $\mathbf{u}$ passes through spatially periodic target. When Bragg conditions are fulfilled two strong waves can be excited in the target. If simultaneously electrons of the beam are under Cherenkov condition, they emit electromagnetic radiation in directions depending on diffraction regime. Case without incident electromagnetic waves corresponds to oscillator generation regime. In our previous work we considered other schemes of VFEL: two-wave Laue geometry, three-wave Bragg-Bragg, Bragg-Laue and Laue-Laue geometry [6]-[8].

Partly, threshold parameters of electron beam instability in VFEL can be investigated by using linear theory. It was shown [9] that in the system it exists several threshold point of the beam current corresponding to beginning of the electron beam instability, regenerative amplification and generation. In our previous works it was shown that variation of VDFB can change the type of generation.

MATHEMATICAL FORMULATION

The system of equations for all cases of VFEL is obtained from Maxwell equations in the slowly-varying envelope approximation using the field representation in the form $\mathbf{E} = \mathbf{e}_i E_i \exp\{i\mathbf{k}_{\tau_i}\mathbf{r} - \omega t\}$, $i = 0, ..., n-1$. Here we restrict ourselves by considering two-wave VDFB. The system for n-wave VDFB can be written by evident generalization. So, we obtain the following nonlinear equations:

$$\begin{aligned}
&\frac{\partial E}{\partial t} + \gamma_0 c \frac{\partial E}{\partial z} + 0.5 i\omega l E - 0.5 i\omega \chi_\tau E_\tau = \\
&2\pi j \Phi \int_0^{2\pi} \frac{2\pi - p}{8\pi^2} \left(\exp(-i\Theta(t,z,p)) + \right. \\
&\left.\exp(-i\Theta(t,z,-p))\right) dp, \\
&\frac{\partial E_\tau}{\partial t} + \gamma_1 c \frac{\partial E_\tau}{\partial z} + 0.5 i\omega \chi_{-\tau} E - 0.5 i\omega l_1 E = 0.
\end{aligned} \tag{1}$$

Here $l_i = \left(k_{\tau_i}^2 c^2 - \omega^2 \varepsilon_0\right)/\omega^2$, $i = 0, 1$, $l = l_0 + \delta$. δ is detuning from exact Cherenkov condition. γ_0, γ_1 are VDFB cosines. $\Phi = \sqrt{l_0 + \chi_0 - 1/(\beta\gamma)^2}$. $\chi_{\pm\tau}$ are Fourier components of the dielectric susceptibility of the target.

System (1) must be supplemented with proper initial and boundary conditions as well as equations for the phase dynamics:

$$\begin{aligned}
&\frac{d^2\Theta(t,z,p)}{dz^2} = \frac{e\Phi}{m\gamma^3\omega^2}\left(k_z - \frac{d\Theta(t,z,p)}{dz}\right)^3 \cdot \\
&\cdot \mathrm{Re}\left(E \exp(i\Theta(t,z,p))\right), \\
&\frac{d\Theta(t,0,p)}{dz} = k_z - \omega/u, \quad \Theta(t,0,p) = p.
\end{aligned} \tag{2}$$

It was proposed in (2) that the electron beam is synchronous with the wave $\mathbf{E}_0$ only. The integral form of beam current in the right hand side of (1) is obtained by averaging over the following initial phases of electrons in the beam: entrance time of electron in interaction zone ωt_0 and transverse coordinate of entrance point in interaction zone $\mathbf{k}_\perp \mathbf{r}_\perp$. Equation (2) depends on these two initial phases only in combination $\mathbf{k}_\perp \mathbf{r}_\perp - \omega t_0$ (that appears in initial condition for phase at $z = 0$). Therefore, in the mean field approximation double integration over two initial phases can be reduced to the single integration. As the result, the averaged current in right hand side of the first equation of (1) differs from expressions frequently used in literature (see e.g.[11]). Method of averaging over initial phases of electrons is well-known [12] and widely used in simulation of BWT (backward wave tube), TWB (travelling wave tube), FEL and other electronic devices. Let us adduce its derivation. We consider magnetized electron beam which propagation can be considered as one-dimensional. The motion equation of one electron in the wave has the next form:

$$\ddot{z} = \frac{e}{m\gamma^3}(\mathbf{e}_\sigma \mathbf{n}) Re\{a \exp(i\mathbf{k}_\perp \mathbf{r}_\perp + ik_z z - i\omega t)\},$$

where e and m are electron charge and mass respectively, γ is the Lorentz factor of electron beam. Initial phase is an individual mark of the electron in beam. Averaging over this phase allows to pass from microscopical description to macroscopical one. Averaging current and applying Liuville's Theorem lead to the following expression:

$$\begin{aligned}
&j \sim j_0 \int d\Theta_0 d\Theta_1 \exp\left\{-i\Theta\left(t, t_0, \mathbf{r}_\perp\right)\right\} = \\
&j_0 \int d\Theta_0 d\Theta_1 \exp\left\{-i\Theta\left(t, \Theta_1 - \Theta_0\right)\right\}
\end{aligned} \tag{3}$$

where $\Theta(t, t_0, \mathbf{r}_\perp) = k_z z + \mathbf{k}_\perp \mathbf{r}_\perp - \omega t(z, t_0)$ is an electron phase. $t(z, t_0)$ is a trajectory of electron emerged at moment t_0 in the target. Initial phase of electron in interaction region has the form:

$$\Theta(t = t_0, t_0, \mathbf{r}_\perp) = \mathbf{k}_\perp \mathbf{r}_\perp - \omega t_0 = \Theta_1 - \Theta_0.$$

Taking into account that the phase depends on initial phases in combination $\Theta_1 - \Theta_0$ only and performing change of variables, we receive term with current in the form of right hand side of (1).

We proposed numerical methods for VFEL modelling [7]. They are implemented in computer code VOLC (VOLume Code)[8]. It was developed on the basis of multiple Fortran codes, created in 1991—2005 years. Dimensionality is 2D (one spatial coordinate and one phase space coordinate) plus time. Different VFEL geometries are investigated in light of experiments on VFEL at INP.

NUMERICAL RESULTS

Let us summarize a great amount of numerical result obtained [6]-[8] and recently. It was investigated:

- generation thresholds subject to beam current, target length, target absorption, diffraction asymmetry factors β for two- and three-wave geometries,
- width of the zone of amplification subject to beam current for two- and three-wave geometries,
- SASE (Self Amplified Spontaneous Emission) regime in Laue and Laue-Laue geometry,
- lasing in Bragg geometry with external mirrors for different reflection coefficients,
- generation regime in Laue geometry with external mirrors,

- different degeneration modes for Bragg-Bragg and Bragg-Laue geometries subject to tuning parameter δ, parameters l_i and diffraction asymmetry factors β_i,
- bifurcation points corresponding to transitions between different regimes of generation for different geometries,
- dependence of position of bifurcation points on geometry of VDBF and other VFEL parameters.

In [10] theoretically derived dependence of the threshold current on asymmetry factor of VDFB was presented. This relation confirms that VDFB allows to control the second threshold current. Numerical results presented in Fig.2 are in close agreement with the theory.

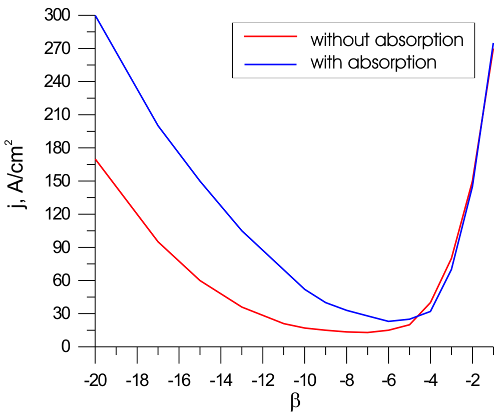

Figure 2: Dependence of the threshold current on asymmetry factor of VDFB

One of the main VFEL physical properties is the following dependence of threshold current in the case of n mode in synchronism for n-wave VFEL [9]: $j_{th} \sim 1/(kL)^{3+2(n-1)}$.

So, threshold current can be significantly decreased when modes are degenerated in multiwave diffraction geometry if $k|\chi|L \gg 1$. On the other hand interaction length can be reduced at given current value. This was confirmed in numerical experiments and depicted in Fig.3.

Last Fig.4 demonstrates attempts to simulate VFEL experiment [13]. Here one can see dependence of the VFEL generation intensity on the length of the "grid" volume resonator with 5 threads in the frame marked with squares and numerically simulated by us dependence of the wave amplitude on the resonator length for the electron beam with the energy 200 keV and current density 2 kA/cm^2.

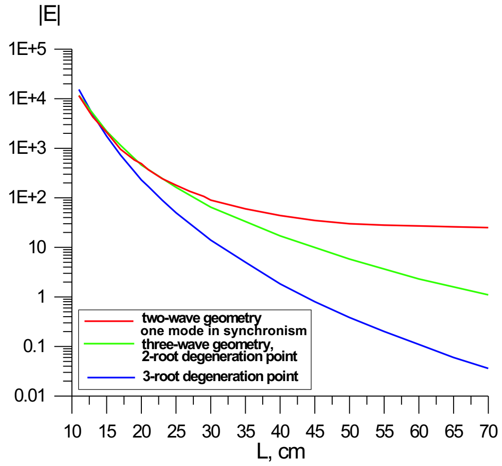

Figure 3: Current threshold for two- and three-wave geometry in dependence on L

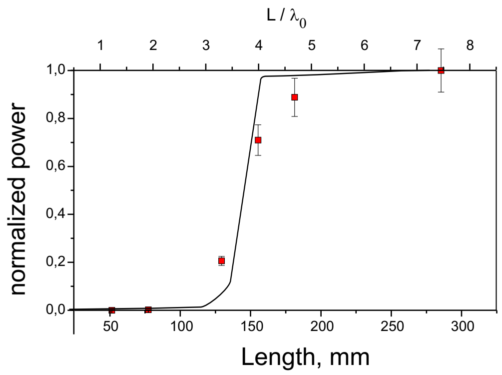

Figure 4: Dependence of electromagnetic radiation on L for experimental setup [13]

CONCLUSION

We developed an instrument to model real VFEL experiments. Mathematical model with computer code VOLC allows to obtain all main VFEL physical dependencies and to investigate the nonlinear stage of its operation.

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