GENERATION REGIMES OF FEL WITH VOLUME DISTRIBUTED FEEDBACK*

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Abstract

This paper discusses different generation regimes of Volume Free Electron Lasers (VFELs) which utilize Volume distributed feedback (VDFB). Dependence of VFEL operation on VDFB parameters is studied.

INTRODUCTION

VDFB significantly extends operation possibilities of Free Electron Lasers (FELs). It was shown in [1] that properly parameters choice of VDFB geometry lowered down the threshold current magnitude of generation. VDFB can solve the problem of mode discrimination at great electromagnetic power generation in oversized systems. The beam cross section in such systems significantly exceeds wavelength. Large sizes of electron beam cross section allow to distribute power and to lower local load of system elements. However, standard oversized system suffers from excitation of great number of parasitic undesired modes. As a result, the destructive interference takes place, the coherence of radiation degrades and the generation efficiency significantly reduces. So, analysis of electron beam dynamics dependence on VDFB parameters is of great importance. Partly, threshold parameters of electron beam instability in VFEL can be investigated by using linear theory. For example, the first threshold point j_1 corresponds to beginning of the electron beam instability. At this point the first radiating mode appears. Instability stage corresponding to regenerative amplification is in the range of parameters $j_1 < j < j_2$ when generating mode is already appeared, but the radiation gain of this mode is less than radiation losses of the coupled feedback mode. Such system can amplify incident wave. Parameters at which radiation gain becomes equal to absorption correspond to the second threshold point. When magnitude of beam current exceeds the second threshold value, generation process occurs without incident wave. For current density j in the range $j_2 < j < j_3$ the beam instability in short period of time changes into the stationary nonlinear saturation regime. If current exceeds the third threshold value $j > j_3$, the nonlinear stage becomes non-stationary. The threshold currents mentioned above depend on parameters of VDFB. So, variation of VDFB can change the type of generation. For providing VFEL experiments and their interpretation it is necessary to have theoretical description and numerical simulation of effects considered. Here some such theoretical results are presented.

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ENERGY TRANSFER IN VDFB SYSTEMS

Let's study energy exchange between electron beam and radiation in the system with VDFB. Electromagnetic field in this case can be expressed as

$$\begin{aligned}\mathbf{E} &= \sum_\alpha a_\alpha \exp\{i\mathbf{k}_\alpha\mathbf{r} - i\omega t\}\{\mathbf{e} + .. \\ &\quad + \mathbf{e}_i s_i \exp[i\tau_i \mathbf{r}] + ..\}, \end{aligned} \tag{1}$$

where τ_i are the set of reciprocal vectors, $\mathbf{e}_i$ are unit polarization vectors, s_i are coupling coefficients between waves, a_α are mode amplitudes which are determined from initial and boundary conditions. Using the equation of electron motion in the electromagnetic field and averaging over initial electron phases the following expression for mean electron energy can be obtained

$$\Delta\gamma = -\frac{e^2(\mathbf{ue})^2}{2m^2u^3c^2\gamma}\left(\frac{k^2}{\omega} - \frac{\mathbf{ku}}{c^2}\right)L_*^3 F, \tag{2}$$

where L_* is the length of interaction region,

$$F = \mathrm{Im}\left[\sum_{\alpha\beta} a_\alpha a_\beta^* \left(\frac{\exp\{i(\nu_\beta - \nu_\alpha)\} - 1}{(\nu_\beta - \nu_\alpha)\nu_\beta^2} + \right.\right.$$
$$\left.\left. + \frac{\exp[-i\nu_\alpha] - 1}{\nu_\alpha \nu_\beta^2} + \frac{\exp[-i\nu_\alpha](i\nu_\alpha + 1) - 1}{i\nu_\alpha^2 \nu_\beta}\right)\right],$$

$\nu_i = (\omega - \mathbf{k}_i\mathbf{u})L_*/u$ are parameters of detuning from synchronism.

Control of Emission Line Shape by VDFB

Generally in the case of n-wave VDFB the energy transfer (2) between electron and electromagnetic field depends on n detuning parameters. This gives additional possibilities to regulate the shape of emission line in comparison with one-wave synchronism case.

Shapes of one-wave and different two-wave synchronism lines is illustrated in Fig. 1. It is seen that corresponding selection of parameter $a = \nu_1 - \nu_2$ increases the width of amplification region by two times (compare the curves "one-wave" and "a=7"). Due to this fact, electron beam will be synchronous with electromagnetic wave for a longer time. So, more energy is transferred to electromagnetic wave that raises the laser efficiency. On the other hand emission line can be narrowed by properly choice of detuning parameters.

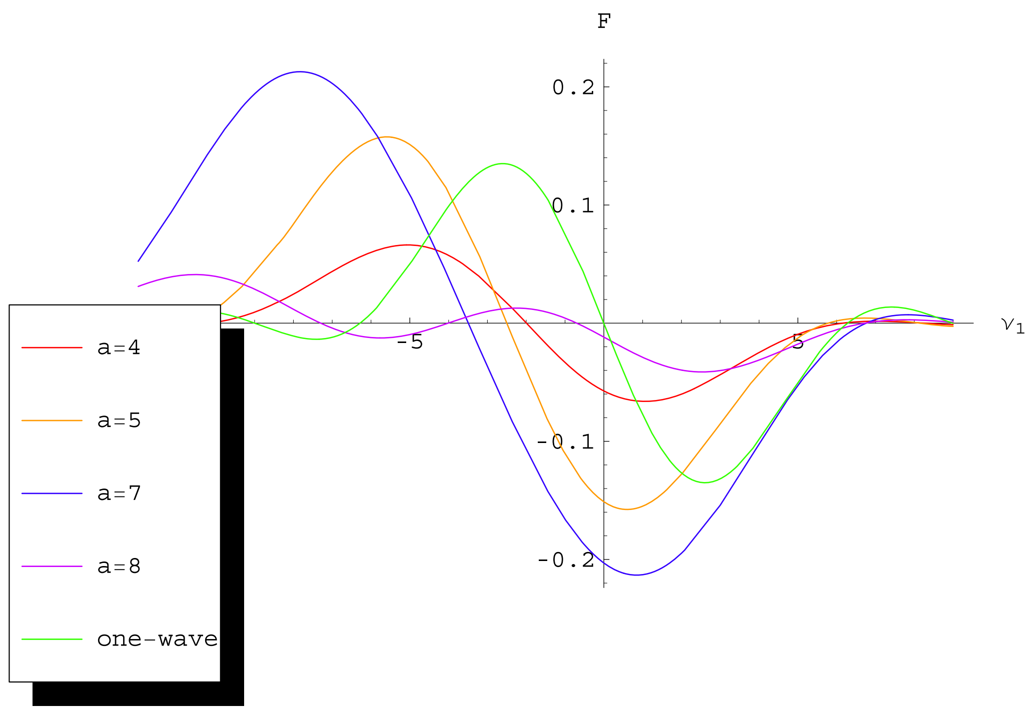

Figure 1: Emission line dependence on the relation between detuning parameters $a = \nu_1 - \nu_2$ for one and two-wave synchronism.

REGENERATIVE AMPLIFICATION (LINEAR STAGE)

Eigenmodes for two-wave VDFB are obtained from the dispersion equation [1]

$$(\omega - \mathbf{ku})^2 \left\{ \left(k^2c^2 - \omega^2\varepsilon_0\right)\left(k_\tau^2 c^2 - \omega^2\varepsilon_0\right) - \omega^4 r \right\} = -\frac{\omega_L^2 (\mathbf{ue})^2}{\gamma}\left(k^2c^2-\omega^2\right)\left(k_\tau^2c^2 - \omega^2\varepsilon_0\right). \tag{3}$$

Here $\mathbf{k}_\tau = \mathbf{k} + \tau$, $\varepsilon_0 = 1 + \chi_0$, $r = \chi_\tau \chi_{-\tau}$, χ_0 and χ_τ are polarizabilities of periodical structure with VDFB which appear in the expansion of permeability:

$$\varepsilon(r, \omega) = 1 + \sum_\tau \chi_\tau \exp\{-i\tau\mathbf{r}\}. \tag{4}$$

Representing solutions of (3) as $\mathbf{k} = \mathbf{k}_0 + \mathbf{n}\delta\omega/c$, where $\mathbf{k}_0$ satisfies to the condition $\omega - \mathbf{k}_0\mathbf{u} = 0$ the following boundary conditions for mode amplitudes a_i can be written:

$$\begin{aligned} \sum_i a_i &= a, \quad \sum_i s_i a_i \exp\{ik\delta_i L\} = b, \\ \sum_i \frac{a_i}{\delta_i} &= 0, \quad \sum_i \frac{a_i}{\delta_i^2} = 0. \end{aligned} \tag{5}$$

The first and the second equalities in (5) correspond to the continuity of electromagnetic field at the input ($z = 0$) and the output ($z = L$) of VDFB structure, a and b are amplitudes of incident waves. The third and the fourth equations correspond to the density and current density continuity of unperturbed at the input electron beam. Solving system (5) one can obtain the following expression for intensity of output electromagnetic waves:

$$\begin{aligned} \frac{I}{I_0} &= \frac{\gamma_0 |\mathbf{E}_0|^2 + \gamma_1 |\mathbf{E}_\tau|^2}{\gamma_0 |a|^2 + \gamma_1 |b|^2} = \\ &= \left(\frac{\gamma_0 c}{\mathbf{nu}}\right)^3 \frac{16\pi^2 n^2}{-\beta\left(k\chi_\tau L_*\right)^2\left(\Gamma_{th} - \Gamma\right)}, \end{aligned} \tag{6}$$

where

$$\Gamma_{th} = \frac{\left(\frac{\gamma_0 c}{\mathbf{nu}}\right)^3 16\pi^2 n^2}{-\beta\left(k\chi_\tau L_*\right)^2 kL_*} + \mathrm{Im}\chi_0\left(1 - \beta \pm \frac{\mathrm{Im}r}{\chi_\tau \mathrm{Im}\chi_0}\right)$$

$$\Gamma = \frac{\pi^2 n^2}{4\gamma}\left(\frac{\omega_L}{\omega}\right)^2 k^2 L_*^2 \left(\chi_0 \pm \sqrt{-\beta}\,|\chi_\tau| - \gamma^{-2}\right)\cdot$$

$$\cdot\left(\chi_0 \pm \sqrt{-\beta}\,|\chi_\tau|\right) f(y),$$

$$f(y) = \sin y \frac{(2y + \pi n)\sin y - y(y + \pi n)\cos y}{y^3 (y + \pi n)^3}.$$

$I_0 = \gamma_0 |\mathbf{E}_0|^2 + \gamma_1 |\mathbf{E}_\tau|^2$ is the intensity of incident waves. $y = \nu_1/2$, β is diffraction asymmetry factor, γ_0 and γ_1 are diffraction cosines. Expression (6) is derived in the low gain approximation.

It follows from (6) that $I/I_0 > 1$ when $\Gamma > \mathrm{Im}\chi_0\left(1 - \beta \pm \mathrm{Im}r/\left(\chi_\tau Im\chi_0\right)\right)$ ($j > j_1$ is the first threshold point) and the amplification process starts. Fig. 2

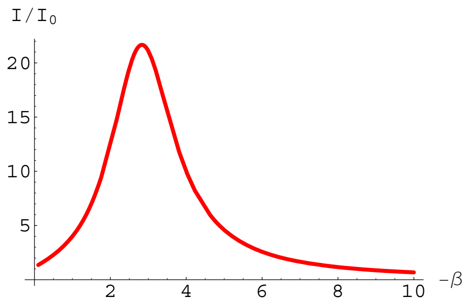

Figure 2: Dependence of regenerative amplification coefficient on asymmetry factor β of VDFB.

demonstrates amplification dependence on VDFB geometry factor β. So, VDFB can control amplification process. When current reaches some critical value $j > j_2$ the oscillation regime develops. Dependence of the second threshold current on VDFB asymmetry is illustrated in Fig. 3. It is seen that VDFB allows to control the second thresh-

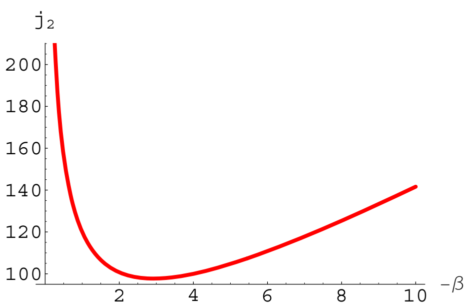

Figure 3: Dependence of the second threshold current j_2 on asymmetry factor β of VDFB.

old current too. Regenerative amplification ($j_1 < j < j_2$) is stationary and can occur in linear or nonlinear regime depending on magnitudes of I (nonlinear regime will be discussed bellow). In contrary, oscillation regime has the non-stationary character and the linear stage rapidly transfers to the nonlinear one. To study nonlinear regime the numerical simulation of VFELs must be carried out.

NUMERICAL SIMULATION OF THE SYSTEM WITH VDFB

For n-wave VDFB from Maxwell equations in the approximation of slow varied amplitudes the system of n equations for n strong coupled waves can be obtained Here we restrict ourselves by considering three-wave VDFB. The system for n-wave VDFB can be written by evident generalization. So, using the field representation in the form $\mathbf{E} = \sum_{\tau_i} \mathbf{e}_i E_i \exp\{i\mathbf{k}_{\tau_i}\mathbf{r} - \omega t\}$, $i = 0, 1, 2$ we obtain the following nonlinear equations:

$$\frac{\partial E_0}{\partial(\omega t)} + \gamma_0 \frac{\partial E_0}{\partial(kz)} + \tfrac{1}{2} l E_0 - \tfrac{1}{2}\chi_1 E_1 - \tfrac{1}{2}\chi_2 E_2$$
$$= 2\pi j \Phi/\omega \int_0^{2\pi} \tfrac{2\pi - p}{8\pi^2} \left(\exp(-i\Theta(t,z,p)) + \right.$$
$$\left. \exp(-i\Theta(t,z,-p))\right) dp,$$
$$\frac{\partial E_1}{\partial(\omega t)} + \gamma_1 \frac{\partial E_1}{\partial(kz)} - \tfrac{1}{2}\chi_{-1} E_0 + \tfrac{1}{2} l_1 E_1 - \qquad (7)$$
$$\tfrac{1}{2}\chi_{2-1} E_2 = 0,$$
$$\frac{\partial E_2}{\partial(\omega t)} + \gamma_2 \frac{\partial E_2}{\partial(kz)} - \tfrac{1}{2}\chi_{-2} E_0 - \tfrac{1}{2}\chi_{1-2} E_1$$
$$+ \tfrac{1}{2} l_2 E_2 = 0.$$

Here $l_i = \left(k_{\tau_i}^2 c^2 - \omega^2 \varepsilon_0\right)/\omega^2$. $\gamma_0,\ \gamma_1,\ \gamma_2$ are three VDFB cosines.

System (7) must be supplemented with equations for the phase dynamics:

$$\frac{d^2\Theta(t,z,p)}{dz^2} = \frac{e\Phi}{m\gamma^3\omega^2}\left(k_z - \tfrac{d\Theta(t,z,p)}{dz}\right)^3 \cdot$$
$$\cdot \mathrm{Re}\left(E_0 \exp(i\Theta(t,z,p))\right), \qquad (8)$$
$$\frac{d\Theta(t,0,p)}{dz} = k_z - \omega/u, \quad \Theta(t,0,p) = p.$$

In (8) it was proposed that the electron beam is synchronous with the wave $\mathbf{E}_0$ only. The integral form of beam current in the right hand side of (7) is obtained by averaging over the following initial phases of electrons in the beam: entrance time of electron in interaction zone ωt_0 and transverse coordinate of entrance point in interaction zone $\mathbf{k}_\perp \mathbf{r}_\perp$.

Equation (8) depends on these two initial phases only in combination $\mathbf{k}_\perp \mathbf{r}_\perp - \omega t_0$ (that appears in initial condition for phase at $z = 0$). Therefore, in the mean field approximation double integration over two initial phases can be reduced to the single integration. As the result, the averaged current in right hand side of the first equation of (7) differs from expressions frequently used in literature (in which the current is used in the form $\int_0^{2\pi} d\Theta_0 \exp\{-i\Theta\}$).

Regimes of VFEL Generation (Nonlinear Stage)

VFEL dynamics simulated on the basis of the system (7) and (8) is illustrated in Fig. 4 - 7. In the Fig. 4 the regimes of regenerative amplification and stationary generation are presented. It follows from this Fig., that the region of regenerative amplification is rather narrow. This region $j_1 < j < j_2$ is marked by two dotted lines. Therefore, the idea to control amplification by regulating VDFB parameter β (and other VDFB parameters) seems to be very useful. Let's note, that the region of regenerative amplification by corresponding choice of VDFB parameters can be

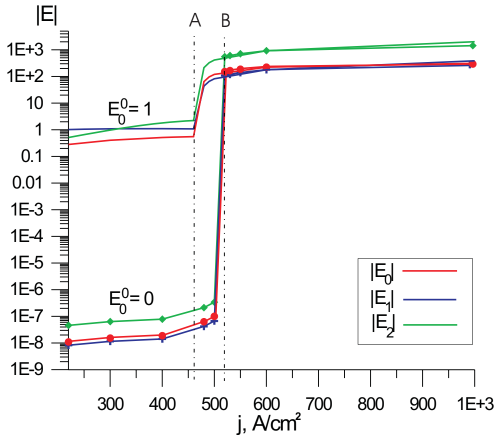

Figure 4: Amplification and oscillation regimes for three-wave VDFB. Vertical dotted lines (A and B) shows positions of the first and the second threshold currents.

increased for three-wave VDFB in comparison with two-wave VDFB ([2]).

When electron beam current increases, generation remains stationary up to the third threshold current j_3. Then at $j > j_3$ non-stationary periodic regime of generation starts. Fig. 5 demonstrates temporal dynamics of generation in this region ($j > j_3$).

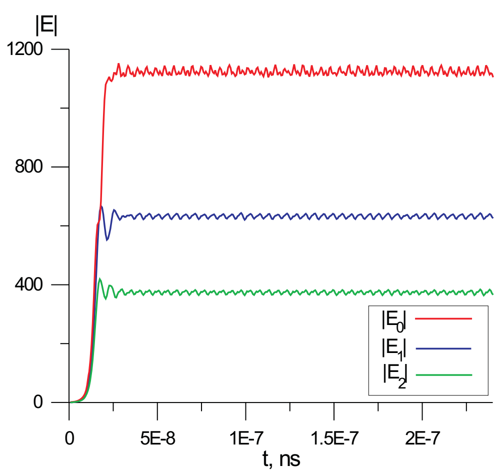

Figure 5: Current exceeds third threshold value ($j > j_3$). Periodic regime of oscillation for three-wave VDFB.

Oscillations appear at discrete number of frequencies at further current increase.

At some current value dynamics becomes chaotic and the regions with continuous frequency spectrum appears (Fig. 6 and Fig. 7).

CONCLUSION

In the paper presented it was shown that VDFB allows to regulate the generation regime. Changing VDFB parameters at given current value can transfer generation process

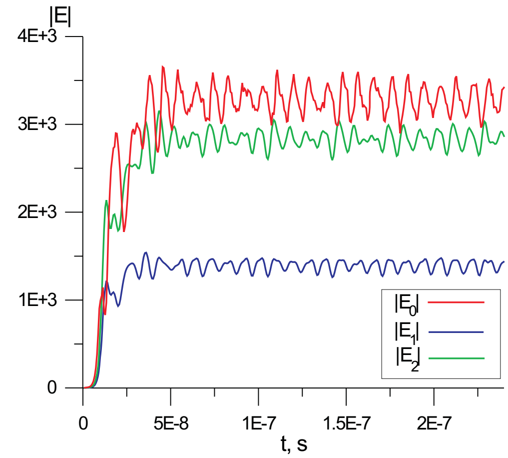

Figure 6: Chaotic oscillation regime.

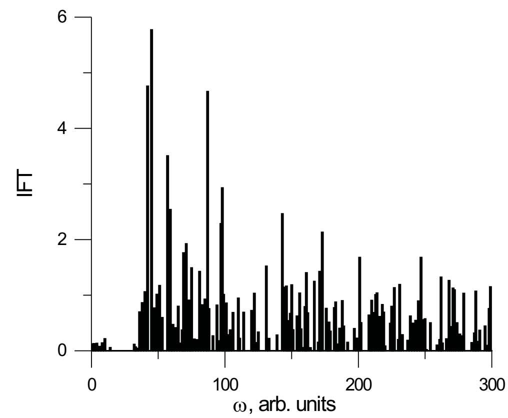

Figure 7: Fourier transform of $\mathbf{E}_0$ corresponding to chaotic generation in Fig. 6 .

between different regimes. In addition n-wave VDFB allows to discriminate parasitic modes at generation of great power in large volume. First experiments with VFEL generation was presented in [3, 4] and the following papers. So, VFEL has great future prospect.

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