On the Simulation of HM-Networks and their Application

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Abstract: *This article describes methods for simulation of HM-networks and the application of these techniques to solve problem of transport logistics.*

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1. GENERAL PRINCIPLES OF SIMULATION QUEUING NETWORKS

Fairly widespread classes of mathematical models are simulation models. These models are implemented as computer programs that step by step reproduce the events occurring in the real system. With regard to their networks simulations reproduce the processes of care applications in the lines of network systems, the transition of applications from a queuing system (QS) to another, waiting in queues for queuing applications, etc. With the simulation it is possible to confirm the computer simulation experiments with precise analytical methods for finding the characteristics of the studied networks, or evaluate the accuracy of approximate methods. The advantage of simulation is that it is applicable to networks of almost any complexity, for which analytical studies very difficult or even impossible. It should be noted, however, that simulation is often required time-consuming.

One type of simulation is a discrete event simulation - simulation of the system at discrete points in time when events occur that reflect the sequence of state changes over time. We consider dynamic or time-varying systems. Therefore, the state system, the properties of the object and the number of active objects, settings, actions and delays - this is a function of time, which is constantly changing during the simulation.

Discrete event simulation can include three different models: an active-oriented model, event-oriented model and process-oriented model.

2. SIMULATION HM-NETWORKS

Consider HM (Howard-Matalytski) - a network of arbitrary structure with the same type of applications, consisting of n queuing systems (QS) $S_1, S_2, \ldots, S_n$. Application for passing from one to other QS brings the latter system, revenue and profit respectively in the first system is reduced by this amount. HM-networks are used as models to predict earnings in banking, information networks, logistic transport systems and other objects [1]. Consider the case where the proceeds of transitions between states of the network are deterministic functions, depending on network conditions and time, and queuing networks are a single line.

Let the network receives a simple stream of similar applications to the intensity of λ, and in QS number of i each application independently of the incoming stream of other applications arrives with probability p_{oi}, $i = \overline{1,n}$, $\sum_{i=1}^{n} p_{oi} = 1$. Application, completing the service in QS number i, immediately and independently of other applications, with probability p_{ij} passes the system S_j, with probability p_{i0} leaves the network, $\sum_{j=0}^{n} p_{ij} = 1$, $i = \overline{1,n}$.

Network states in this case given by the vector $k(t) = (k_1(t), k_2(t), \ldots, k_n(t))$, where $k_i(t)$ – number of requests in QS of number i at time t, $i = \overline{1,n}$.

Let's designate through $v_i(k,t)$ – the full expected income which is received by system S_i in time t, if during the initial moment of time the network is in state k; $r_i(k)$ – the income of system S_i in unit of time, when the network is in state k; I_i – vector with dimension n with zero components, behind an exception component with number i, which is equal 1; $r_i(k)$ - the income of system S_i, when the network makes transition from state (k,t) to state $(k + I_i, t + \Delta t)$ during time Δt; $-r_i(k)$ – the income of this system if the network makes transition from state (k,t) to state $(k - I_i, t + \Delta t)$; $R_{ij}(k)$ – the income of system S_i, (the expense or loss of system S_j), when the network changes state from (k,t) to $(k + I_i - I_j, t + \Delta t)$ during Δt, $i, j = \overline{1,n}$.

Process-oriented model are used for simulate the HM-networks. It is based on the use of objects of two types: 1) the active type - processes, 2) non-active types - resources.

Processes (active objects) include the application served on the network that correspond to reality, for example, customers in the bank, packages on a computer network. The resources include a variety of queuing network, which correspond to the booths in the banks, urban exchanges, network file servers, etc.

We construct a simulation model of changes in income in systems closed HM-network. We consider the objects of these two types. Every QS is a resource, and each application is served by the network - a process. Each resource has a set of properties. These properties include the servicing capacity, which equals the number of service lines in the QS, the distribution of service times of requests in each line. Processes are not special properties.

Necessary to consider the objects of another type, called a generator of income. Random variables R_{ij}, r_i, $i=\overline{1,n}$ are reimages of the generators of income in the model, in a simulation model of the proceeds of transitions between states of the network may be random. Each QS should be compared to the two generators of income - the first to get the value of the gains from the transition from the application of the QS to another and the second - for winning the QS over the time interval during which all bids were at the service.

Since the processes are active objects, they can "grab" resources when they are free, "hold" them for some time, or stand in line to resources, which is equivalent to service requests in the queuing system. After the capture and retention of a single resource, the process proceeds to a different resource, according to the matrix of transition probabilities. During the transition from one resource to another, a certain income moves from resource to resource, source-receiver. This value is determined by the generator, the associated resources involved in the transition. Similarly, can be constructed a simulation model changes in income in an open HM-network. It suffices to introduce two additional resource - the resource generator (to generate new processes with given intensity) and the storage resource (for the reception process, leaving the network). In addition, the model is necessary to add additional types of income generators.

We used the package SimPy (Simulation in Python) - an open software library for simulation of different processes with discrete events, written in the programming language Python. This fully object-oriented library provides developers with all the necessary tools to develop simulation models of almost any complexity. It allows the simulation of different processes in parallel, and also allows us to describe the logic of interaction between processes. An important feature of the library is to use the generator language Python, one of the characteristics of which are possible to interrupt their work, and then continue from where it functioning.

To present the simulation components in the package, there are two main classes - Process for the active entities, processes, groups of entities shared processes and Resource - for resources. In addition, introduced an important class of Monitor, which collects the simulated data for analysis and statistics. SimPy package was taken as the basis for the implementation of simulation models of changes in expected revenues in HM-network in the event of different types of income from the transitions between its states. Were calculated by means of library various operating parameters investigated networks were used to explore changes in expected income, depending on the time.

3. APPLICATION OF SIMULATION HM-NETWORK FOR PROBLEMS TRANSPORT LOGISTICS

Economic, economic-mathematical, statistical methods for solving theoretical and practical problems are usually used in logistics. Flows of products coming from manufacturers to recipients at random times, random length of time intervals required to produce the product, its loading and unloading location for warehouses and sales, the need to use predetermined methods of queuing theory to develop mathematical models of the functioning of logistics transport systems (LTS).

Clearly, if we want to describe the operation of different manufacturers, warehouses and consumers as a single system, the model can serve as a functioning network of Defense, consisting of various QS corresponding to these subjects. For the LTS objectives are important task of evaluating and predicting earnings of their subjects, derived from the transport of products between different modes of transport.

Suppose that in the LTS consists of n subjects S_1, S_2, …, S_n (it may be manufacturing plants, warehouses and sales points), between which transportation is carried out of goods. Transportation of goods by car from one subject to another gives him a certain income derived from sales and income of another entity is reduced by some amount. The costs of transportation (fuel, car repairs), the driver's salary may be classified as a first or second subject. Times' unloading - loading "cars in the subjects of LTS and streams of cars between them - are random. It is necessary to estimate (predict) the expected (average) income from such subjects LF transport. A similar situation arises when evaluating and predicting earnings of transport company (TC), carrying freight between the subjects. Shipping from subject S_i to subject S_j brings some TC, in general, perquisite, but TC is the material loss. HM-networks are used to solve these problems.

Example 1. Consider a closed Markov HM-network with the same type, consisting of $M=n+m_1+\ldots+m_{n-1}$ service systems S_i, $i=1,..n,1_1,\ldots,1_{m_1},\ldots,(n-1)_1,\ldots,(n-1)_{m_{(n-1)}}$, shown in Figure 1, which is a model of a transportation of goods.

In this model system S_n - a "producer", which produces a product, system S_1, S_2, …, S_{n-1} - "storage" in which the storage of the product, S_{i_1}, S_{i_2}, …, $S_{i_{m_i}}$ - "shops" (points of sale of goods) which comes from the warehouse S_i, $i=\overline{1,(n-1)}$.

At the same time under the application refers to the carriage of goods in the logistics system, "producer - warehouses - items the sale of goods".

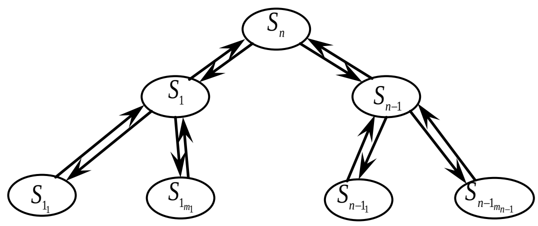

Fig. 1 - Network model of transporting goods

Consider the application of this model in a particular case. We describe the operation of a particular transport company (TC), which transports goods. The main activity company "Dezhits Ya.Yu." (Grodno) is road freight transport. The company has in stock a few trucks refrigerated to 22 ton. The majority of the goods carried are food, perishable goods and goods for which transportation will need a special temperature mode. His company provides services to dozens of different organizations, among them company "Karat-Expedition", ODO "Dominikastroy", SLC "ABC Company", LLC "Leminvest", etc.

In analyzing the statistical data the company obtained from data on the traffic logs, number of customers and applications for a flight in 2009, have been found appropriate intensity of care (Table 1). Under the service rate is the amount of vehicle trips company "Dezhits Ya.Yu." to carry out relevant flights per unit time (per month).

Table 1. Data on completed flights company "Dezhits Ya.Yu."

Period	The intensity service	Period	The intensity service
January	8	July	4
February	7	August	4
March	10	September	6
April	8	October	13
May	6	November	7
June	5	December	5

Using the software Advanced Grapher 2.2 and statistics company "Dezhits Ya.Yu.", has been found best approximating function for the intensity of service requests

$$\mu_n(t) = 3{,}671\cdot 10^{-4} t^8 - 0{,}019t^7 + 0{,}397t^6 - 4{,}55t^5 - \\ -121{,}47t^3 + 274{,}763t^2 - 314{,}882t + 143{,}25 \qquad (1)$$

We describe how to use the technique discussed above in forecasting revenues of TC. Suppose, for example, $M = 3$, $K = 4$, $\mu_1 = 5{,}0$, $\mu_{1_1} = 1{,}5$, $\mu_2(t)$ has the form (1). Transport scheme is shown in Figure 2.

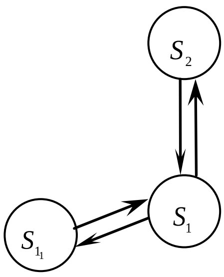

Fig. 2 - The scheme of transporting goods in the three systems of care

Under the system, "shipper's warehouse" shall mean a warehouse shipper, such as SLC "ABC Company", "Baltika" (shown in Fig. 2, this system S_1), and under the "consignee" - a warehouse of the consignee for example by "NP-Tools', LLC "Evroopt" and other companies (in Fig. 2 this system S_{1_1}), the system S_2 - it's TAs.

Assume that the proceeds of state transitions do not depend on the states and time. Matrix of transition probabilities between the QS network applications and single-step income during the transition between the QS applications have the form

$$P = \|p_{ij}\|_{3\times 3} = \begin{pmatrix} 0 & 0{,}4 & 0{,}6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

$$R = \|r_{ij}\|_{3\times 3} = \begin{pmatrix} 3 & 1{,}2 & 3{,}2 \\ 1{,}8 & 5{,}1 & 0 \\ 0 & 2{,}5 & 1{,}9 \end{pmatrix}.$$

Because the network is closed, the number of states $l = C_{3+4-1}^{3-1} = 15$.

Consider the case where service times of applications in systems S_1, S_2 that are normally distributed with mean 2, 4 and variance 1, 3, and in S_{1_1} the exponential distribution with parameter 3. We define the vector of initial states $V(0) = (0,0,\ldots,0)$. With the help of simulation results are obtained (Figure 3).

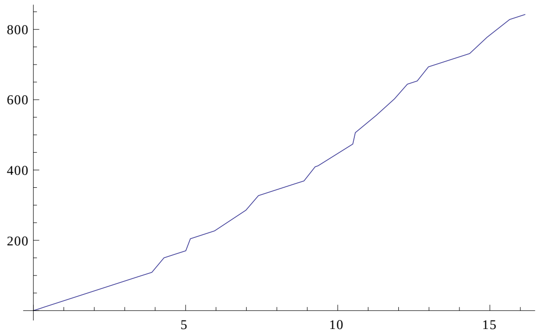

Fig. 3 - Changing the income of S_1

Consider the case where service times of applications in systems S_1, S_2 S_{1_1} the exponential distribution with parameter 2, 4, 3 respectively. We define the vector of initial states $V(0) = (0,0,\ldots,0)$.

With the help of simulation results are obtained (Figure 4, line 1). We also obtain the expected returns using analytical methods [2] (Figure 4, line 2).

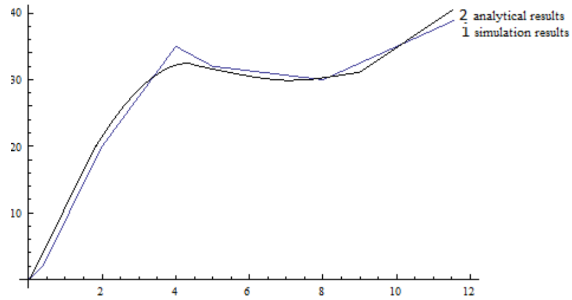

Fig. 4 - Changes in income of S_2

4. REFERENCES

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