Stochastic Modeling for Choice of Tariff Policy in Corporate Local Network

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Abstract: *In article a stochastic model of university local network (LAN) functioning as queueing network (QN) with one-line systems is investigated. Optimization problems for finding an optimal traffic policy and optimal channel capacity which minimizes customer's fee for communication service during given time interval are formulated. Solution methods for formulated problems are observed.*

Keywords**:** Local area network, queueing network, optimal channel capacity, optimal tariff policy.

1. INTRODUCTION

The modern level of scientific researches and market economy development assumes fast growth of systems of collection, storage and information allocation. The industry of LAN manufacture developed with amazing speed for the last years. Implementation of LAN is based on increase of efficiency and productivity of staff in the core. This purpose is proclaimed by developers and firms-suppliers of LAN and administration of different institutions.

One of the real ways of solution the problem of improvement of quality of LAN functioning is wide implementation of methods of mathematical modeling among which the leading position is occupied with methods of the theory of QN. QN are used for determination of the major system characteristics of LAN: productivity; time of delivery of packets; probabilities of loss of messages and lock in nodes; areas of admissible values of loading at which demanded service quality, etc. Thus, the usage of the QN theory allows to receive analytical models for solution of a wide range problems of LAN research.

2. DESCRIPTION OF UNIVERSITY LAN AND SUBSTANTIAL STATEMENT OF PROBLEM

The LAN of Grodno state university is constructed on technology Ethernet with usage of a fiber optic cable and unites all educational buildings of university. The various departments which information flows are tightly interconnected by the nature of the activity function at the university. There are problems of information dialogue as inside user groups which are territorially located on small distance, and between user groups. Working groups are united by means of LAN and have possibility to exchange the data as inside, and between groups. LAN control, routing and a filtration of information flows is carried out by means of computers and the appropriate software.

For connection of university LAN with Internet three proxy servers are used:

-proxy.grsu.by – access to the Internet without authorization with a speed to 300 Kbit/sec;

-proxy1.grsu.by – services the university departments;

-proxy2.grsu.by – services the faculties.

Calls of external users from Internet to resources of LAN servers: web-server, mail server, proxy-server, and calls of internal users to Internet resources create the total load in communication lines of four Internet-providers, which have specified order channel capacity and which is defined by agreement between LAN administration and providers. Finding of optimal channel capacity and optimal tariff plan of Internet-providers are urgent problems. There are two cases can be observed: 1) if high channel capacity is ordered but network idles then LAN administration incurs unreasonable high loss and coasts of LAN resources should be reduced; 2) if low channel capacity is ordered and network is overload then waiting time of service startup by users is big and users incurs unreasonable high loss with low coasts of LAN resources therefore coasts of LAN resources should be enhanced.

3. STOCHASTIC MODEL OF LAN

To build the mathematical model of LAN we will use elements of the queueing theory. Models of functioning of network servers are one-line queueing systems (QS).

Let's describe the messages in such models. We will consider, for example, the proxy-server which receives requests from LAN users for access to the Internet resources, executes request availability of this resources and implements requests after receiving positive reply. The information exchange between the proxy-server and the Internet resources is carried out by means of package switching (the information is transferred by packets with usage of the installed protocols of communication during certain sessions). Thus each request generates set of additional sessions which named the generated sessions. If LAN user has addressed, for example, to a site, each additional session serves normally for transmission of the objects allocated on a site (the pictures, separate frames, banners etc.) in the form of separate files. Set of the generated sessions of one LAN user is the integral message.

Distribution law of intervals between arrival time of requests describes the type of input flow of messages in QN. Service of each message consists in file transfer of defined size V_i with transfer rate G_i, which equals to server capacity . The set of random variables (RV) $\eta_i = \dfrac{V_i}{G_i}$ generates the set of service time and their distribution law describes the service type. For statistical analysis of distribution law of service time of requests on the proxy-server during the control moments samplings of the requests of sizes $V_1, \ldots, V_n$ were observed, n = 64209. On the basis of the given sampling the variation series for RV V which equals to size of request incoming to proxy-server and RV η which equals service time of request in proxy-server was constructed. On the basis of a variation series for η empirical distribution function (d.f.) $F_e(t)$ was constructed. Its plot during a control interval 8.00-18.00 is presented on fig. 1. Plots of empirical d.f. have a similar type in remaining checkpoints of observation. The exponential character of service times distribution is visible from plot $F_e(t)$. Therefore as the theoretical distribution of service times the exponential distribution $F_t(t)=1-e^{-\mu t}$ is accepted, where μ – parameter of exponential distribution, $\mu=\frac{1}{\bar{\eta}}$, where $\bar{\eta}$ – mean service time of one request.

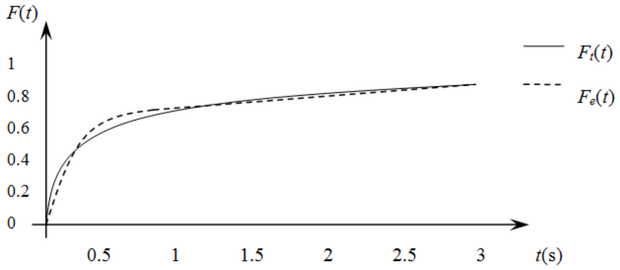

Fig. – Plots of empirical and theoretical d.f.

The hypothesis about the exponential distribution law of service times was checked in all checkpoints with help of criterion "chi-square" at a significance value $\alpha=0.05$. Calculations by means of function ХИ2ТЕСТ built in a packet of Microsoft Excel of 2007, have shown that results of observations of RV η don't contradict a hypothesis about exponential character of service times distribution.

4. PROBLEM OF CHOICE OF OPTIMAL TARIFF PLAN FOR LAN

Today there is a great number of the Internet-providers developing and implementing new tariff model, capable to satisfy needs of a wide range of clients and giving a freedom in choosing of the suitable tariff plan. Tariff plan – the standard form of the offer in which the prepaid traffic and transfer rate of the data, and also their cost is underlined. Tariff plans are divided on tariff with not limited access (without the prepaid traffic), tariff with the prepaid traffic and tariff plans of an hourly access to the Internet. If user connects the tariff of not limited network access to the Internet he gets unlimited access to the Internet resources for fixed fee defined in the contract with the provider, and the size of a fee depends on the selected data transmission rate (Kbit/sec). If user connects the tariff with the prepaid traffic he pays a monthly fee depending on the selected traffic, and all traffic used over prepaid, is paid for other tariff (sometimes depending on day time). The unused prepaid traffic isn't transferred to the next month. If user connects the tariff plans of an hourly access to the Internet then the monthly fee depends on selected time of using the Internet and data transmission rate.

Let's consider the optimization problem for finding of the optimal tariff plan for university LAN. It is obvious that it is necessary to consider the following characteristics during tariff plan choice: monthly consumed traffic, day times of most heavy use of the Internet, data transmission rate.

The main loading on exterior communication channel of university Internet-providers is formed by work of proxy-server, therefore we will analyze only the traffic transiting through this server. To calculate the amount of the downloaded (transferred) information during time T we will use expression

$$V_{\inf}^{l}=\int_{0}^{T}\bar{V}_l\,\bar{N}(t)dt,\ l=\overline{1,2},$$

where $\bar{V}_l$ – average amount of download (transferred) information in time unit, l – traffic identifier (l=1 corresponding to incoming traffic, l=2 corresponding to outgoing traffic); $\bar{N}$ – mean value of requests at server in a time unit. The tariffing of the traffic downloaded over the ordered tariff plan, depends on day time, so for calculation of cost of this traffic it is necessary to calculate amount of the downloaded information during those time intervals. The total traffic, downloaded over norm during time T, equals

$$\bar{G}=\sum_{l=1}^{2}\sum_{j=1}^{k}\int_{\Omega_j}\bar{V}_l^{\,j}\,\bar{N}^{j}(t)dt-G,$$

where G – prepaid traffic, $\bar{V}_l^{\,j}$ – average volume of download (transferred) information in a time unit in j-th tariff interval, $\bar{N}^{j}$ – mean value of requests at server in a time unit in interval j – one of the time intervals of tariff plan, $j=\overline{1,k}$, k – count of time intervals with different coast of traffic in chosen tariff plan, Ω_j – summary time of using the Internet in j-th tariff interval during time T, i.e. $T=\bigcup_{j=1}^{k}\Omega_j$. Let's consider that traffic $\alpha_j^l\bar{G}$ is received over the norm in j-th time interval, where $\alpha_j^l=\frac{\bar{V}_l^{\,j}}{\bar{V}_l}$ are weight coefficients which satisfied to condition $\sum_{l=1}^{2}\sum_{j=1}^{k}\alpha_j^l=1$, $l=1,2$, $j=\overline{1,k}$.

Optimization problem looks as follows

$$\begin{cases} W=C(G)+\sum_{l=1}^{2}\sum_{j=1}^{k}g_j^l\alpha_j^l\left(\int_{\Omega_j}\bar{V}_l^{\,j}\,\bar{N}^{j}(t)dt-G\right)\to\min_{G}, \\ G<\sum_{j=1}^{k}\int_{\Omega_j}\bar{V}_l^{\,j}\,\bar{N}^{j}(t)dt<2G, \end{cases}\qquad(1)$$

where W – fee for provider service, g_j^l – coast of the traffic unit which was downloaded (transferred) over prepaid traffic in j-th tariff interval, $C(G)$ – coast of chosen tariff plan. Restriction of the given problem is formulated taking into account that the total consumed traffic shouldn't be less than prepaid traffic, but also not to exceed it more than twice as in this case it is more favorable to use a tariff with a prepaid traffic from above $2G$.

In case when user costs depends on data transition rate and doesn't depend on traffic, the optimization problem is

$$\begin{cases} W = C(V) \to \min\limits_V \\ \sum\limits_{i=1}^{n} \overline{V_i}\,\overline{N_i} \le V, \end{cases} \qquad (2)$$

where V – channel capacity, $C(V)$ – coast of tariff plan which depends on rate of data transfer, restriction of represented optimization problem is formulated taking into account that amount of download information in a time unit shouldn't exceed the rate of channel capacity. Both described optimization problems can be solved by method of full search of tariff plans of chosen Internet-provider: tariff plans with different prepaid traffic in problem (1), tariff plans with different rate of data transfer in problem (2).

5. PROBLEM OF FINDING THE OPTIMAL CHANNEL CAPACITY OF LAN AND METHODS OF ITS SOLUTION

Let's consider problem of choice of optimal LAN channel capacity

$$\begin{cases} S_{net}(G,T) = C(G)\dfrac{1}{GT}\displaystyle\int_0^T \sum_{j=1}^{N_{ser}} MV_{ij} Mv_j(G,t)dt \to \min_G, \\ g_1 \le G \le g_2, \end{cases} \qquad (3)$$

where $S_{net}(G,t)$ – fee of provider service, $C(G)$ – coast of traffic unit which is function of channel capacity G Kbit/sec and presents the maximum count of kilobytes in a second which dedicated channel able to transfer, V_{ij} – size (in bytes) of i-th message at j-th server, $v_j(G,t)$ – count of messages transferring via j-th server at time t, N_{ser} – count of servers, symbol M means expected value.

Let's mark that university LAN functions generally in the conditions of high loading, i.e. there are requests to servers at any moment. This results from the fact that LAN users can leave requests for downloading of Internet files on workplaces for the period of their absence, and also create loading at university servers by means of remote access. Therefore mean value of messages $Mv_j(G,t)$, transiting through j-th server at the moment t, can be found by using method based on applying of multidimentional generating functions [1]. The given method is just intended for finding of time-dependent characteristics of models in such conditions.

The problem (3) is the minimization problem at interval of function of one variable G which has difficult enough appearance. It can be presented in the form of multiple power series; solution of the equation $S'_{net}(G,t) = 0$ is difficult. However it is known that in practice parameter G usually is an integer value, therefore it is possible to consider the problem (3) as the problem of integer programming which can be solved by means of combinatorial methods of discrete programming. In particular it is possible to use a method of branches and bounds which uses the serial-to-parallel circuit for building of a tree of possible variants. The algorithm for solution of the optimization problem (3) by means of branch and bounds method with a simple and effective method of an estimation of upper bound of objective function is described in [2].

6. CONCLUSION

Optimization problems of finding the optimal tariff plan and optimal channel capacity of Internet-provider which reduce coasts of service are formulated. The methods of their solution are observed. Let's present some range of application of results: estimation of total coast for using Internet services; finding the optimal tariff plan providing by Internet-providers; forecasting coasts in case of situation change (closing of tariff plan, opening of new tariff plans and bonuses), etc.

6. REFERENCES

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