Assignment Process for the Optimization of the Products Distribution in the Commercial Centres

A.M. Gil Lafuente [1], Anna Klimova [2]

1) University of Barcelona, Diagonal 690, 08034, amgil@ub.edu, www.ub.es

2) University of Barcelona, Diagonal 690, 08034, aklimova@ub.edu, www.ub.es

Abstract: *The objective of this article was to show the importance of the correct choice of the assignment method in the commercial process under uncertainty. The globalization process and the current economic crisis have forced the companies to make the profound changes in their management policies and strategies and also to base their decision making process on the flexible instruments that can consider any relevant information. We developed a study case using Hungarian Algorithm. This algorithm allows obtaining an optimization in the assignment process between two groups of variables according to some common characteristics.*

Keywords**:** Assignment Process, Hungarian Algorithm, Uncertainty, Distribution Process.

1. INTRODUCTION

Nowadays the question of survival of a company often depends on the correct and rapid use in its business of the modern mathematical tools of uncertainty adapted to the conditions of instability and permanent changes. The process of globalization and economic crisis are the two main reasons that caused the situation of uncertainty in the world society [1,2] and forced the companies to make the profound changes in their management policies and strategies.

This complex situation requires that the small and large companies change their actions to guarantee the adequacy of the decision making process in a new economic environment.

These approaches are adapted to all areas of a business that pretends to achieve its objectives with a minimal chance for success. These measures affect the areas beginning from the housekeeping department to the marketing or human resources departments, which put all the emphasis on managing the production process in a modern enterprise. By this manner the logistics became a highlight point in the company management assuming all the importance for company's daily performance [3,4].

In this paper we consider only one particular aspect of the logistics such as the assignment of alimentary products in the commercial centers. The company that is under commercial expansion and consolidation process supposes to optimize the assignment process of some of its products that sells in different areas of the city. In order to find an optimal solution for this problem it was used the Hungarian Algorithm that distinguishes among other assignment methods for its great theoretical and practical interest [5]. The idea of this method is to find the assignments (matches) so that to achieve an optimization process using two sets of elements interconnected between each other by a predetermined matrix. This method is characterized by its operational simplicity and also it bases on a multivalent logic that represents a key aspect for the analysis of future scenarios in a financial environment affected by the changes.

In the first part of this work it is highlighted the importance of the impact on the economy and business management produced by the globalization process and the economic crisis. A brief analysis of the development of the assignment methods is made. The importance of Hungarian Algorithm in optimization of the assignment problems under uncertainty is highlighted. A study case is developed. Hungarian Algorithm was used.

2. EFFECTS OF THE COMMERCIAL GLOBALIZATION AND ECONOMIC CRISES

Globalization is an historical process which refereed mainly to the commercial interchange between different international economies.

The process of globalization started in the decade of the 60s when companies in search of new markets extended their scope of actions beyond national borders, passing from being the producers in their countries of origin to the establishment of its manufacturing plants in the foreign countries [6,7].

From the decade of the 80s the links between different international economies started to tighten having as a result the participation of foreign shareholders in the companies. This phenomenon increased due to the transnational corporate mergers and corporate investment that took place in the 90s of last century [8-10].

The most noticeable effects of globalization are the following: the growing spending on goods and services made by the national citizens due to the large volume of import and, on the other hand, the increasing volume of export of domestic products under the continuous and increasingly varied exchange between different countries. This is the scenario where a systematic logistics activity obtains the maximum importance becoming a very useful tool in the economic relations [3,11-13].

Another no less important subject is a current financial crisis. The crisis causes havocs in all sectors of the world economy. The devastating effect of this phenomenon has attacked mainly the activity of small and medium enterprises.

Nowadays the economic turbulence and the situation of uncertainty occur constantly in the market. This is characterized by a constantly changing environment which affects the way the companies perform their activity as well as by the fact of permanent technological revolution which make everything become obsolete in a short time. Despite of this worrying perspective some businesses manage not only to survive but emerge from the crisis stronger increasing volumes of their sales.

According to the numerous studies made in the recent years the following conclusion was obtained: the key variable for the growth of a business consist in a global change of its strategy and management. It is necessary to focus on the innovation, on the satisfaction and customer loyalty, on new marketing strategies, internationalization and fusion [3,14].

3. LOGISTICS SISTEM IN THE ACTUAL ENVIRONMENT

Nowadays a term known as logistics has begun to be used in business since 1960s when the National Council of Physical Distribution Management began to use it to define this new branch of industry. This concept can be determined as follows:

"A set of activities that deals with the total-flow of materials that starts with the supply of raw materials and ends with the delivery of finished products to the customers. The logistics management is responsible for planning, performance and monitoring all activities related to the total-flow through the company of raw materials, components, intermediate products and finished products, and all the information associated to it" [11-13].

There are some reasons that have motivated the selection of logistics as one of the essential competitive factor for the performance of the companies [3,12,13]:

- A lifecycle of the products shortens continuously, so the management of the goods stock has become a key element for combining the production and the optimal distribution of the products in a perfect symbiosis. Thus the risk of obsolescence increases gradually and logistics functions are of great importance.
- Increase of the references in a company's catalogue. Thus the functions of the logistics are to make records of the goods, to manage the client's orders, to administrate warehouses and manufacturing flows of the goods.
- A decisive word in a decision making process of a logistics chain depends on the intermediates and customers that is because of the changes produced in a distribution system with the appearance of the wholesale distribution and big supermarkets.
- Globalization of the markets has removed the barriers between countries.
- Information technology advances have been implemented efficiently in a real-time control of goods movement and relationships between different stages of distribution channels.

These factors together with the fact that the logistics costs form up to 25% of the total cost of an international trade transaction, have made the logistics to be an object of a big concern for the companies.

4. DEVELOPMENT OF THE ASSIGNMENT METHODS. HUNGARIAN ALGORITHM IN THE ECONOMIC SHERE OF THE COMPANY

The expression "assignment problem" (AP) has first appeared in 1952 in the paper of Votaw and Orden [15], but as the beginning of the wide development of practical solution methods and the variations on the classic assignment problem was the publication of Kuhn's article on the Hungarian method for its solution in 1955 [5,16]. The Kuhn's Hungarian method has become the starting point of a fast developing area such as Combinatorial Optimization.

AP is looking for an optimal matching of the elements of two or more sets, where the dimension of the problem refers to the number of sets of elements to be matched. Originally, the assignment problem involved assigning each task to a different agent, with each agent being assigned at most one task.

AP problems can be categorized into linear, quadratic, bottleneck, multidimensional, etc. The AP can also be represented in a large number of forms such as mathematical programming, combinatorial, or graph-theoretic formulations, and constitute one of the most important objects in computer science, operations research, and discrete mathematics [17]. There are numerous applications of the AP in other disciplines of science and engineering such as chemistry, physics, electrical engineering, etc. [18-20].

Over the past 50 years a large number of variations on the classic assignment problem has been proposed.

The linear assignment problem, for example, is one of the fundamental models in operations research, computer science, and discrete mathematics [17]. It involves the optimal matching of *n* workers to *n* jobs that has the lowest total cost, if the cost of assigning worker *i* to task *j* equals c_{ij}. The LAP is also included as a basic part in other optimization problems, such as quadratic assignment problem, multidimensional assignment problem, travelling salesman problem [21], etc.

The objective of the bottleneck AP is to minimize the maximum of the costs of the assignments, while the classic AP tries to minimize or maximize the sum of the costs of the assignments of tasks to agents [21,22].

In many cases there are multiple decision criteria and it is important to find a solution that recognizes all of them. The multiple criteria are recognized in decision models either by combining them into a single criterion or by considering them separately and sequentially [23].

The robust assignment problem is looking for an optimal solution for the assignment problem under uncertainty [24].

In some cases, however, the problem needs to match the members of three or more sets. This type of situations is called as multi-dimensional assignment problems [25,26].

There are different variants of multi-dimensional assignment problems such as the tree-dimensional bottleneck assignment problem [27], multi-period assignment problems [28], etc.

A significant contribution in solving the assignment problems in the area of human resource management was made by J. Gil-Aluja [29]. The objective is to find the optimal assignment of the people (potential candidates) to different jobs offered by a company. To solve this problem the author highlights the Hungarian algorithm as a more adequate and optimal tool.

J. Gil-Aluja uses the same methodology to find the optimal solutions for the assignment problems in marketing and logistics areas [30-32]. A.M. Gil Lafuente made a great contribution in the financial area [33,34].

5. THEORETICAL ASPECTS OF HUNARIAN ALGORITHM

The basic objective of the Hungarian algorithm consists of looking for the process of optimization basing on the two sets of elements that are related between themselves through the known matrix.

If the optimization process is based on the minimization principle it is necessary to start from the matrix of distances, $\left[\underset{\sim}{P}'\right]$, for example. In this case, it is very common to have the rectangular matrices to use.

To start the process the matrix $\left[\underset{\sim}{P}'\right]$ is converted into matrix $\left[\overline{\underset{\sim}{P}'}\right]$. There are the following steps [33,34]:

1) We take the first column (P_1) and subtract the smallest element from each element of the same. In each box we will have:

$$P_{ij}^{(1)} = P_{ij} - \min_i P_{ij}$$

This process is repeated for each of the remaining column so that one zero at least appears in each one of them.

We then do the same operations for each one of the rows so that once again at least one zero appears in each one. We then arrive at:

$$P_{ij}^{(1)} = P_{ij} - \min_j P_{ij}$$

2) Once we have arrived at matrix $\left[\underset{\sim}{B}\right]$ we check to see if it is possible to make an assignment in which the values $P_{ij}^{(1)}$ of the solution are all zeros. If this is so then we have arrived at the optimum. On the contrary the process is continued by means of the following steps:

a) The rows containing the least zeros are taken one by one.
b) One of the zeros of the considered row is framed and the remaining zeros that are members of the same are crossed out, as well as the zeros in the column of which the framed zero is a member.
c) This process is repeated for each row until there are no zeros to frame.

3) Obtain the least number of rows and columns that contain all the zeros; in order to do this we must proceed with the following steps:

a) All the rows in which there are not framed zeros are signalled with an arrow.
b) The columns in which a crossed out zero does exist in a row with an arrow are signalled.
c) The columns in which a squared zero does exist in a column signalled with an arrow are likewise signalled. This process is repeated until no further rows or columns can be signalled.
d) Finally a line is drawn through the rows that are not signalled by arrows and through the columns that are signalled with arrows. These rows and columns constitute the least number that possess framed or crossed out zeros.

The rows and columns through which a line has been drawn constitute the so-called “minimum support” of matrix $\left[\underset{\sim}{B}\right]$.

4) The lowest value is taken from among the elements of the matrix that have not been lined. The figure that is taken is subtracted from the elements of the columns that are not lined and this is added to the rows that are lined.

5) With this new matrix the process is restarted at point 2, by continuing on with the same operations used for the previous matrix. In the event of arriving at an optimum solution the process is stopped; to the contrary continue the process from points 3 and 4.

6. STUDY CASE

6. 1. Description of the problem

The company that is under commercial expansion and consolidation process supposes to optimize the assignment process of some of its products that sells in different areas of the city. 6 zones are considered that is equivalent to 6 commercial centers.

First, the potential and the main features of all areas of the city are studied. There are 9 characteristics that correspond to demographic and geographic variables.

The company basing on its long experience in the food industry, constant changes in consumer preferences and taking into account the increasing volatility of the demand wants to offer the customers the products that are demanded more depending on the characteristics of the district.

In order to get an optimal assignment of products to each zone the Hungarian Algorithm is used. Let’s start from the information presented below.

6.2. Description of the input data

The supermarkets (areas) are represented by the set *S*:

$$S = \{S_1, S_2, S_3, S_4, S_5, S_6\}$$

where S_1 - bedroom areas; S_2 - universities/ colleges areas; S_3 - residential areas; S_4 - old town area; S_5 - small business areas; S_6 - great trade areas.

These zones have the following characteristics:

$$C = \{C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9,\},$$

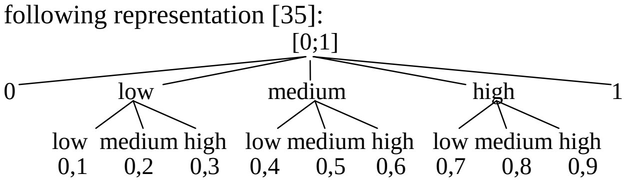
where C_1 - average population age; C_2 - average family size; C_3 - economic level; C_4 - population density; C_5 - % migrant population; C_6 - % average unemployment level; C_7 -educational facilities; C_8 -historical/ cultural activities; C_8 – commercial activities.

The company wants to introduce 8 types of products typologies:

$$P = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9,\},$$

where P_1 -based on fruits, vegetables, meat and fresh fish; P_2 -based on fast and precooked food; P_3 - based on exotic products fresh/ canned; P_4 - based on dairy/ bakery/ churrería; P_5 - based on fresh and packaged products; P_6 - based on fresh and dairy products; P_7 - based on processed, precooked and frozen products; P_8 - based on processed and precooked products, bakery/dairy products.

Regarding the characteristics and in order to establish the relationship between supermarkets and typologies of products, the endecadaria scale is defined semantically. The scale is generalized for 9 features. It has the following representation [35]:



We have the following relationship between the indicators such as *S* and *C*:

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
S_1	0,5	0,3	0,5	0,8	0,6	0,5	0,3	0,2	0,2
S_2	0,3	0,3	0,4	0,8	0,2	0,4	0,9	0,6	0,1
S_3	0,6	0,9	0,3	0,3	0,1	0,1	0,5	0,3	0,5
S_4	0,5	0,5	0,4	0,7	0,5	0,2	0,5	0,8	0,9
S_5	0,6	0,8	0,3	0,4	0,2	0,3	0,4	0,4	0,7
S_6	0,5	0,7	0,4	0,5	0,3	0,2	0,3	0,4	0,8

By the same manner we have the relationship between the indicators such as P and C:

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
P_1	0,5	0,3	0,6	0,6	0,3	0,2	0,2	0,3	0,7
P_2	0,2	0,4	0,3	0,7	0,5	0,3	0,7	0,8	0,8
P_3	0,4	0,3	0,5	0,6	0,6	0,2	0,2	0,6	0,7
P_4	0,4	0,4	0,4	0,7	0,4	0,4	0,4	0,5	0,7
P_5	0,4	0,3	0,5	0,6	0,3	0,3	0,2	0,3	0,6
P_6	0,3	0,4	0,4	0,6	0,4	0,3	0,3	0,4	0,6
P_7	0,3	0,4	0,4	0,5	0,4	0,3	0,4	0,6	0,7
P_8	0,4	0,4	0,4	0,6	0,4	0,3	0,5	0,6	0,7

The valuations assigned to S and P represent the levels of maximum acceptance.

6.3. Obtaining the "coefficient of qualification"

As a next step a first approach called "coefficient of qualification" is made. It is based on the progressive acceptance of those characteristics that correspond to the products typologies which valuations remain on the threshold established by the supermarkets. If the feature overpasses the established level then this feature is penalized. There are:

$\mu_{C_b}(P_i) \in [0;1]$ - the value of the membership function related to the product typology P_i and characteristic C_i.

$\mu_{C_b}(P_j) \in [0;1]$ - the value of the membership function related to the supermarket S_j and characteristic C_j.

We have:

1) If $\mu_{C_b}(P_i) \leqslant \mu_{C_b}(S_j)$ then the characteristic is accepted with the valuation determined as $(1-\mu_{C_b}(S_j))$.

2) If $\mu_{C_b}(P_i) > \mu_{C_b}(S_j)$ then the characteristic is accepted with the valuation as $\overline{1 \wedge (\mu_{C_b}(S_j) + \mu_{C_b}(P_i))}$.

Let's compare S_1 and P_1 in relation to each characteristic. We have:

C_1: $\mu_{C_1}(P_1) \leqslant \mu_{C_1}(S_1)$ as $0,5 \leqslant 0,5$ then $(1-\mu_{C_1}(S_1)) = (1-0,5) = 0,5$;…

C_9: $\mu_{C_9}(P_1) > \mu_{C_9}(S_1)$ as $0,7 > 0,2$ then $\overline{1 \wedge (\mu_{C_9}(S_1) + \mu_{C_9}(P_1))}$ $\overline{1 \wedge (0,2+0,7)} = \overline{0,9} = 0,1$.

Let's calculate a weighted sum of the obtained valuations for each feature:

$$Q(P_1;S_1) = \frac{0,5(+)0,7(+)0,1(+)0,2(+)0,4(+)0,5(+)0,7(+)0,5(+)0,1}{9} = 0,411$$

Let's compare S_2 and P_1 in relation to each characteristic. We have:

C_1: $\mu_{C_1}(P_1) > \mu_{C_1}(S_2)$ as $0,5 > 0,3$ then $\overline{1 \wedge (\mu_{C_1}(S_2) + \mu_{C_1}(P_1))}$ $\overline{1 \wedge (0,3+0,5)} = \overline{0,8} = 0,2$.

Let's calculate a weighted sumo of the obtained valuations for each characteristic:

$$Q(P_1;S_2) = \frac{0,2(+)0,7(+)0,2(+)0,2(+)0,5(+)0,6(+)0,1(+)0,4(+)0,2}{9} = 0,344$$

The qualification coefficients for the rest of the supermarkets are the following:

$Q(P_1;S_1) = 0,411$;… $Q(P_2;S_1) = 0,289$;

$Q(P_3;S_1) = 0,389$;… $Q(P_4;S_1) = 0,322$;…

$Q(P_5;S_1) = 0,433$;… $Q(P_8;S_6) = 0,267$.

6.4. Assigning of valuations to the characteristics

In order to estimate the characteristics in relation to each supermarket it is necessary to normalize the valuations established for the purpose to make a convex weight. Let's summarize the matrixes of the previous part into one matrix $[\underset{\sim}{P}]$.

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
S_1	0,5	0,3	0,6	0,6	0,3	0,2	0,2	0,3
S_2	0,2	0,4	0,3	0,7	0,5	0,3	0,7	0,8
S_3	0,4	0,3	0,5	0,6	0,6	0,2	0,2	0,6
S_4	0,3	0,4	0,4	0,6	0,4	0,3	0,3	0,4
S_5	0,3	0,4	0,4	0,5	0,4	0,3	0,4	0,6
S_6	0,4	0,4	0,4	0,6	0,4	0,3	0,5	0,6

This matrix shows the adequacy of the products typologies to the needs of supermarkets. The closer the valuations are to 1 the greater is the level of similarity.

The characteristics have a different importance for the supermarkets when they assess the products to be sold. These weights in the interval [0;1] are established.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
$\underset{\sim}{S}_1$	0,6	0,5	0,8	0,7	0,3	0,4	0,3	0,2	0,4
$\underset{\sim}{S}_2$	0,7	0,5	0,7	0,8	0,3	0,3	0,6	0,4	0,3
$\underset{\sim}{S}_3$	0,5	0,6	0,7	0,8	0,2	0,4	0,3	0,2	0,3
$\underset{\sim}{S}_4$	0,4	0,3	0,5	0,7	0,3	0,2	0,5	0,9	0,9
$\underset{\sim}{S}_5$	0,6	0,4	0,6	0,2	0,2	0,4	0,3	0,6	0,8
$\underset{\sim}{S}_6$	0,7	0,3	0,5	0,8	0,3	0,5	0,3	0,8	0,9

Let's normalize the data of the previous matrix. We have the following matrix $[\underset{\sim}{E}]$:

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
S_1	0,14	0,12	0,19	0,17	0,07	0,10	0,07	0,05	0,10
S_2	0,15	0,11	0,15	0,17	0,07	0,07	0,13	0,09	0,07
S_3	0,13	0,15	0,18	0,20	0,05	0,10	0,08	0,05	0,08
S_4	0,09	0,06	0,11	0,15	0,06	0,04	0,11	0,19	0,19
S_5	0,15	0,10	0,15	0,05	0,05	0,10	0,07	0,15	0,20
S_6	0,14	0,06	0,10	0,16	0,06	0,10	0,06	0,16	0,18

Let's convert each matrix $[\underset{\sim}{P}_i]$ into vector. For this purpose the product $[\underset{\sim}{E}] \cdot [\underset{\sim}{P}_i]$ is found (i=1,...8) by taking one element of $[\underset{\sim}{E}]$ and multiplying it for one element of $[\underset{\sim}{P}_i]$. The products are summed to each row. By this manner the vector is obtained for each matrix $[\underset{\sim}{P}_i]$.

We collocate all the products into one matrix $[\underset{\sim}{P}']$.

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
S_1	0,367	0,350	0,371	0,319	0,395	0,362	0,314	0,307
S_2	0,304	0,328	0,315	0,300	0,341	0,367	0,361	0,300
S_3	0,293	0,233	0,265	0,258	0,300	0,293	0,303	0,283
S_4	0,306	0,304	0,285	0,311	0,304	0,315	0,315	0,315
S_5	0,390	0,234	0,302	0,302	0,405	0,415	0,332	0,290
S_6	0,396	0,231	0,276	0,251	0,376	0,363	0,324	0,239

This matrix is taken as a basis for the development of the assignment model.

6.5. Hungarian Algorithm

Let's find a matrix $[\overline{\underset{\sim}{P}'}]$ that is complementary to $[\underset{\sim}{P}']$.

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
S_1	0,633	0,650	0,629	0,681	0,605	0,638	0,686	0,693
S_2	0,696	0,672	0,685	0,700	0,659	0,633	0,639	0,700

S_3	0,708	0,768	0,735	0,743	0,700	0,708	0,698	0,718
S_4	0,694	0,696	0,715	0,689	0,696	0,685	0,685	0,685
S_5	0,610	0,766	0,698	0,698	0,595	0,585	0,668	0,710
	0,604	0,769	0,724	0,749	0,624	0,637	0,676	0,761
F_1	1	1	1	1	1	1	1	1
F_2	1	1	1	1	1	1	1	1

1) Following the process described in the part 5.1 we obtain the matrix $\left[\underset{\sim}{B}\right]$:

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
S_1	0,029	0	0	0	0,010	0,053	0,047	0,008
S_2	0,092	0,022	0,056	0,019	0,064	0,047	0	0,015
S_3	0,104	0,118	0,106	0,062	0,105	0,122	0,058	0,032
S_4	0,090	0,046	0,086	0,008	0,101	0,100	0,046	0
S_5	0,006	0,116	0,069	0,017	0	0	0,029	0,025
S_6	0	0,119	0,095	0,068	0,028	0,052	0,037	0,076
F_1	0,396	0,350	0,371	0,319	0,405	0,415	0,361	0,315
F_2	0,396	0,350	0,371	0,319	0,405	0,415	0,361	0,315

2) Following the process described in the part 5.2 we have the following results:

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
S_1	0,029	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	0,010	0,053	0,047	0,008
S_2	0,092	0,022	0,056	0,019	0,064	0,047	0	0,015
S_3	0,072	0,085	0,074	0,030	0,073	0,090	0,026	$\cancel{0}$
S_4	0,090	0,046	0,086	0,008	0,101	0,100	0,046	$\boxed{0}$
S_5	0,006	0,116	0,069	0,017	$\cancel{0}$	$\boxed{0}$	0,029	0,025
S_6	$\boxed{0}$	0,119	0,095	0,068	0,028	0,052	0,037	0,076
F_1	0,081	0,035	0,056	0,004	0,090	0,100	0,046	$\cancel{0}$
F_2	0,081	0,035	0,056	0,004	0,090	0,100	0,046	$\cancel{0}$

3) We follow the process described in the part 5.3. As a result we have:

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
S_1	~~0,029~~	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	~~0,010~~	~~0,053~~	~~0,047~~	~~0,008~~
S_2	0,092	0,022	0,056	0,019	0,064	0,047	$\boxed{0}$	0,015
S_3	0,072	0,085	0,074	0,030	0,073	0,090	0,026	$\cancel{0}$
S_4	0,090	0,046	0,086	0,008	0,101	0,100	0,046	$\boxed{0}$
S_5	~~0,006~~	~~0,116~~	~~0,069~~	~~0,017~~	$\cancel{0}$	$\boxed{0}$	~~0,029~~	~~0,025~~
S_6	$\boxed{0}$	~~0,119~~	~~0,095~~	~~0,068~~	~~0,028~~	~~0,052~~	~~0,037~~	~~0,076~~
F_1	0,081	0,035	0,056	0,004	0,090	0,100	0,046	$\cancel{0}$
F_2	0,081	0,035	0,056	0,004	0,090	0,100	0,046	$\cancel{0}$

The rows and columns crossed by a line represent a "minimum support" of the matrix $\left[\underset{\sim}{B}\right]$.

4) We follow the steps described in the part 5.4. The smallest element of the matrix is 0,004. As a result we have the following matrix:

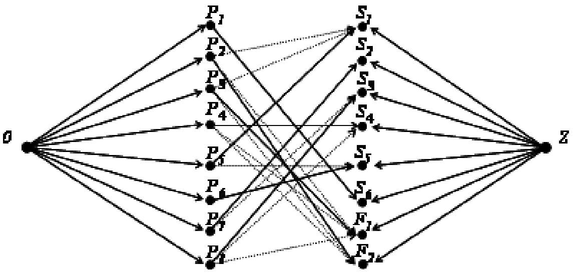
	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
S_1	0,029	0	0	0	0,010	0,053	0,047	0,012
S_2	0,092	0,022	0,056	0,019	0,064	0,047	0	0,019
S_3	0,068	0,081	0,070	0,026	0,069	0,086	0,022	0
S_4	0,086	0,042	0,082	0,004	0,097	0,096	0,042	0
S_5	0,006	0,116	0,069	0,017	0	0	0,029	0,029
S_6	0	0,119	0,095	0,068	0,028	0,052	0,037	0,080
F_1	0,077	0,031	0,052	0	0,086	0,096	0,042	0
F_2	0,077	0,031	0,052	0	0,086	0,096	0,042	0

5) The process of the parts 5.2, 5.3 and 5.4 are repeated to obtain an optimal solution.

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
S_1	~~0,029~~	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	~~0,010~~	~~0,053~~	~~0,047~~	~~0,012~~
S_2	~~0,092~~	~~0,022~~	~~0,056~~	~~0,019~~	~~0,064~~	~~0,047~~	$\boxed{0}$	~~0,019~~
S_3	0,068	0,081	0,070	0,026	0,069	0,086	0,022	$\cancel{0}$
S_4	0,086	0,042	0,082	0,004	0,097	0,096	0,042	$\boxed{0}$
S_5	~~0,006~~	~~0,116~~	~~0,069~~	~~0,017~~	$\boxed{0}$	$\cancel{0}$	~~0,029~~	~~0,029~~
S_6	$\boxed{0}$	~~0,119~~	~~0,095~~	~~0,068~~	~~0,028~~	~~0,052~~	~~0,037~~	~~0,080~~
F_1	0,077	0,031	0,052	$\boxed{0}$	0,086	0,096	0,042	$\cancel{0}$
F_2	0,077	0,031	0,052	$\cancel{0}$	0,086	0,096	0,042	$\cancel{0}$

To continue the process the smallest element 0,022 is taken. We repeated the part 5.2 except the parts 5.3 and 5.4. We obtained the optimal solution. The final matrix and associated graph are shown below:

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8
S_1	0,019	$\cancel{0}$	$\cancel{0}$	0,031	$\boxed{0}$	0,043	0,060	0,047
S_2	0,058	0,009	0,022	0,037	0,030	0,013	$\boxed{0}$	0,041
S_3	0,012	0,046	0,014	0,022	0,013	0,030	$\cancel{0}$	$\boxed{0}$
S_4	0,030	0,007	0,026	$\boxed{0}$	0,041	0,040	0,020	$\cancel{0}$
S_5	0,006	0,126	0,079	0,058	$\cancel{0}$	$\boxed{0}$	0,042	0,064
S_6	$\boxed{0}$	0,129	0,105	0,109	0,028	0,052	0,050	0,115
F_1	0,015	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	0,024	0,034	0,024	0,004
F_2	0,015	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	0,024	0,034	0,024	0,004



We found that all products typologies are assigned to the supermarket (area). We have:

1) $P_1 \to S_6$; 2) $P_2 \to F_2$; 3) $P_3 \to F_1$; 4) $P_4 \to S_4$;
5) $P_5 \to S_1$; 6) $P_4 \to S_5$; 7) $P_7 \to S_2$; 8) $P_8 \to S_3$

The products P_2 and P_3 are discarded because the supermarkets F_1 and F_2 are fictitious.

The minimum distance provided by the assignment process is the following:

$$d(P_j, S_i) = d(P_1, S_6) + d(P_4; S_4) + d(P_5; S_1) +$$
$$+ d(P_6; S_5) + d(P_7; S_2) + d(P_8; S_3) =$$
$$= 0,444 + 0,367 + 0,433 + 0,40 + 0,367 + 0,30 = 0,385$$

7. CONCLUSIONS

The objective of this study was to show the importance of the correct choice of the assignment method in the commercial process in an environment of permanent changes.

In the first section the strong influence of the globalization process and the current economic crisis on the activities of modern enterprises are highlighted. These two reasons have forced the companies to make the profound changes not only in their management policies and strategies but also to be more flexible and to adapt more quickly to the market permanent changes.

In this context, the decision making process must be based on the flexible instruments that can consider any relevant information. In this sense, the Hungarian Algorithm allows to obtain an optimization in the assignment process between two groups of variables according to some common characteristics.

The obtained results show the best possible alternative of the assignment process, taking into account the relative importance that the described characteristics have for each of the variables. Furthermore, the assignment takes into account the overall weight of all characteristics, such that, when there are several possible assignments, it is looking for the combination that optimizes all the assignments considered together.

This model is more efficient when a greater number of the input elements are considered.

To resume, the use of the Hungarian Algorithm for the assignment of the products typologies is very effective in an environment in which economic and social evolution provokes the continuous changes. The use of the flexible and adaptive mathematical tools improves, with the low costs, the decision making process of the business.

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