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NONLINEAR REFRACTION AT GAIN SATURATION IN QUANTUM WELLS *

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Peculiarities of nonlinear refraction in the active layer of quantum-well heterolasers are established. Effects of spectral broadening are taken into account, the nonlinear refraction spectrum is calculated, and the nonlinear refraction coefficient is estimated in the heterostructures in the GaAs–AlGaAs system.

Keywords: quantum-well laser, gain saturation, nonlinear refraction

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When studying optical properties of quantum-well (QW) semiconductor structures it is necessary to take into account dispersion of light and the nonlinear response of the active medium, including effects of gain saturation and nonlinear refraction [1–4]. The account of the nonlinear refraction effects is important for selecting optimum conditions of harmonic generation, parametric excitation of radiation, four-wave mixing and other nonlinear optical processes.

Previously, exciton effects in the index of refraction of multiple-QW heterostructures and composite superlattices have been considered [5] and the measured optical nonlinear parameters of multiple-QW heterostructures were compared with the bulk semiconductor data [6]. The basic laws of absorption saturation and nonlinear refraction in two-dimensional semiconductor systems are established in [7, 8]. Features of nonlinear optical processes in doped superlattices are described in [9].

In the present work, effects of nonlinear refraction at the gain saturation are considered as a result of the population change of the subband energy levels in QW heterostructures. Calculations are based on the Kramers–Krönig relation for the GaAs–AlGaAs laser system, and the dependence of the refractive index on light intensity is established.

In QWs in the case of direct optical transitions between electron and hole subbands, the $\mathbf{k}$-selection rule is also supplied with the conservation condition of the subband number. Taking the degree of excitation of a QW heterostructure as a difference of the quasi-Fermi levels ΔF and summing up over all various optical transitions at a light frequency ν, it is possible to calculate the gain spectrum $k(\nu)$ and its change at the excitation of nonequilibrium charge carriers [10]. The calculations are carried out in the effective mass approximation taking into account anisotropy of the effective masses of holes in the QW layer.

The character of spectral broadening at the radiative transitions is very important. Scattering of quasiparticles on phonons and with each other reduces the lifetime of nonequilibrium charge carriers. The basic role is played by collisions hole-to-hole, electron-to-hole, and hole-to-longitudinal optical phonon, therefore, the broadening parameter of the spectral line Γ_{cv} is actually determined by time constants of intraband relaxation of electrons and holes [11].

Changes in the refractive index at the excitation of the active region of a QW heterostructure will be determined for the stationary conditions. A similar approach was used for bulk crystals [2] and applied to two-dimensional semiconductor systems [7–9, 12].

In this case, the rate equation has the form [2]

$$\frac{\eta' j}{ed} = \frac{R_{\rm sp}}{\eta_{\rm sp}}\left[1 - \exp\left(-\frac{\Delta F}{kT}\right)\right] + vk\frac{U}{h\nu_{\rm exc}}, \qquad (1)$$

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where j is the current density in the QW structure, η' is the injection efficiency, d is the QW width, $R_{\rm sp}$ is the spontaneous recombination rate, $\eta_{\rm sp}$ is the quantum efficiency of spontaneous emission, ΔF is the difference of the quasi-Fermi levels in the QW, T is the crystal temperature, v is the group velocity of light, k is the gain coefficient, and U is the radiation density at the excitation frequency $\nu_{\rm exc}$. The spontaneous recombination rate $R_{\rm sp}$ is determined as the integration sum of the spectral spontaneous emission rate $r_{\rm sp}(h\nu)$ over all photon energies $h\nu$. Here, it is supposed, as a rule, that $h\nu \gg kT$. The rate $r_{\rm sp}(h\nu)$ is simple related to the gain $k(\nu)$.

When taking into account the spectral broadening effects, the gain coefficient is described by the expression [10]

$$k(\nu) = \frac{4\pi A_{cv}}{h^2 v \rho d} \sum_i \sum_n m_{{\rm rt}i} \times \int \left[1 - \exp\left(\frac{E_{cv} - \Delta F}{kT}\right)\right] f_{\rm e}(E_{cni}) \times f_{\rm h}(E_{vni}) L(h\nu - E_{cv}) \alpha_{ni}(E_{cv})\, {\rm d}E_{cv}. \quad (2)$$

Here, A_{cv} is the Einstein coefficient for spontaneous recombination transitions, $\rho(h\nu)$ is the density of electromagnetic modes in the crystal, $n = 1, 2, \ldots$ is the subband quantum number, $i = {\rm h}, {\rm l}$ is the index of heavy and light holes, $m_{{\rm rt}i}$ is the reduced mass with taking into account the transverse component of the effective mass of heavy or light holes, $E_{cv} = E_{cni} - E_{vni}$ is the energy of optical transitions, $f_{\rm e}(E_{cni})$ and $f_{\rm h}(E_{vni})$ are the Fermi–Dirac functions for electrons and holes. Quantum-size energy levels, between which the optical transitions occur, E_{cni} and E_{vni}, are determined by the ground states E_{cn} and E_{vn} and by dispersion of the energy spectrum in the subbands of electrons and holes.

The lower limit of the integration in Eq. (2) is taken to be equal to the energy of initial transitions through the ground heavy hole subband $h\nu_{\rm 1h} = E_{\rm g} + E_{c1} + E_{v{\rm h}1}$. In the general case, the energy bandgap $E_{\rm g}$ can be a function of the level of excitation of the active region. The upper integration limit is restricted to the potential barrier height since the filling of high lying energy states during the excitation of the heterostructure changes weakly and optical transitions via them have a bulk character.

As a form of the spectral broadening function $L(h\nu - E_{cv})$, an asymmetric shape of the emission line is taken, part of which at $h\nu - E_{cv} \geq 0$ is represented by the Lorentzian function and another one at $h\nu - E_{cv} < 0$ is represented by the Gaussian [10]. Moreover, the equality is supposed of the spectral broadening parameters $\Gamma_{cv} = \sigma_{cv}$, which determine half-width Γ_{cv} and $(\ln 2)^{1/2}\sigma_{cv}$ of the appropriate tails of the spectral broadening form function.

The quantity $\alpha_{ni}(E_{cv})$ characterizes polarization dispersion of the optical transition probability and depends on the type of mode (TE or TM), kind of holes (heavy or light), and the optical transition energy [13]. The polarization factor $\alpha_{ni}(E_{cv})$ is determined by orientation of the electric field of the light wave and of the wave vector of holes with respect to the dimensional quantization axis [10, 12–14].

Depending on the excitation conditions of the active region, the nonlinear refraction is related to the absorption saturation, where $\Delta F < h\nu_{\rm exc}$ [7, 8], or to the gain saturation, where $\Delta F > h\nu_{\rm exc}$ [12]. Further, the second case will be considered, where the semiconductor in the initial state is excited, for example, by electric current. Therefore, the quantity ΔF is maximum ($\Delta F = \Delta F_0$) at a given j and in the absence of light flow ($U = 0$) and approaches $h\nu_{\rm exc}$ with the growth of optical excitation.

Increase of the optical excitation power results in changes in the gain spectrum and, accordingly, in the refractive index of the active region. Connection between the changes in the gain coefficient Δk and refractive index Δn is described by the Kramers–Krönig relation [8, 15]:

$$\Delta n(\nu_0) = \frac{c}{2\pi^2} \int_0^\infty \frac{\Delta k(\nu)\, {\rm d}\nu}{\nu^2 - \nu_0^2}. \quad (3)$$

Here the integral is taken in the principal value sense. The change $\Delta n(\nu_0) = n(\nu_0) - n_0(\nu_0)$ is determined at a fixed frequency ν_0, and $\Delta k(\nu) = k_0(\nu) - k(\nu)$ covers the whole spectrum of possible frequencies ν. Initial values k_0 and n_0 correspond to the absence of optical excitation ($U = 0$), i. e. to the initial condition, where $\Delta F = \Delta F_0$.

Numerical calculations of the characteristics related to the nonlinear refraction were performed for the QWs of widths of 5 and 20 nm in the GaAs–$Al_{0.3}Ga_{0.7}As$ system. It was assumed that $T = 300$ K, $E_{\rm g} = 1.42$ eV, $\Gamma_{cv} = 10$ meV. By integrating Eq. (3), the values of the integrand function at energy points from the singular point with the step of 0.001 eV up to the height of potential barriers are first calculated and then they are summed up. Reduction of the step down to 0.0005 and doubling the upper limit of integration have

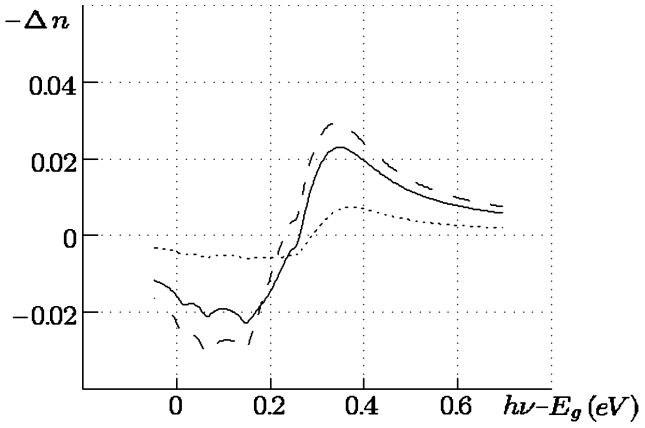

Fig. 1. Spectra $\Delta n(\nu)$ for the TM mode at the gain saturation: $vU = 1.6{\cdot}10^2$ (dotted curve), $9.1{\cdot}10^2$ (solid curve), and $5.1{\cdot}10^3$ kW/cm^2 (dashed curve). $d = 20$ nm, $h\nu_{\rm exc} = 1.524$ eV, $\Delta F_0 = 1.67$ eV.

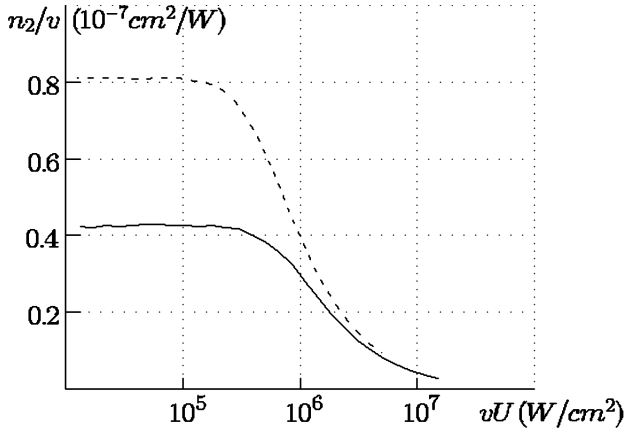

Fig. 2. The dependence of the nonlinear refraction coefficient n_2 versus the density of radiation U in the absence of spectral broadening (dashed curve) and in the case of the asymmetric broadening form function with the parameter $\Gamma_{cv} = 10$ meV at the gain saturation (solid curve). $d = 5$ nm, TE mode, $h\nu_{\rm exc} = 1.524$ eV, $h\nu = 1.55$ eV.

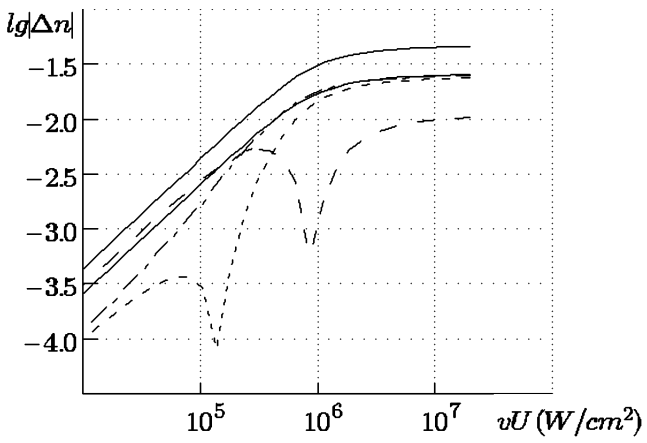

Fig. 3. The dependence of the change in the refractive index Δn on the density of radiation U for various photon energies $h\nu$: 1.464 eV (bottom solid curve), 1.544 eV (top solid curve), 1.664 eV (dashed curve), 1.704 eV (dotted curve), 1.744 eV (broken curve). $d =$ 5 nm, TE mode, $h\nu_{\rm exc} = 1.524$ eV.

in fact no influence upon the results of the calculations.

As shown earlier, the change in the refractive index Δn with increasing the radiation density U in the active region of the undoped QW heterostructure at the absorption saturation ($j = 0$) follows practically a degree function with the power of $1/2$ [16]. For the conditions of the gain saturation, a linear dependence $\Delta n(U)$ is characteristic (Fig. 1). Assuming $\Delta n = n_2 U$, one obtains the nonlinear refraction coefficient n_2. In particular, the resonance increase in the refractive index at the radiation frequency ($\nu = \nu_{\rm exc}$) in the gain interval is determined by the quantity n_2/v of the order of $4{\cdot}10^{-5}$ cm^2/kW. Then, the real part of the third-order complex susceptibility $\chi^{(3)}$ of the QW amplifying medium is estimated [15]:

$$\chi^{(3)} = \frac{cn_0^2}{2\pi}\frac{n_2}{v}.$$

Using $n_0 \approx 3.5$, one obtains $\chi^{(3)} \approx 3{\cdot}10^{-9}$ cm^2/V^2 which is by magnitude several times less than for the QW absorbing medium [8].

According to the obtained relation between Δn and U, the nonlinear refraction coefficient remains practically constant up to the levels of excitation of about 200 kW/cm^2 (Fig. 2). Under these conditions, the maximum change in the refractive index does not exceed 10^{-2}. With the increase of the light flow density vU over 200 kW/cm^2, the nonlinear refraction coefficient decreases as well as the module $|n_2|$ at the absorption saturation. However, the change in the refractive index attains the level of about $5{\cdot}10^{-2}$ for U values one order of magnitude higher than in the case of the absorption saturation [16].

Change in Δn at different frequencies with the growth of the density of radiation U for the undoped active region at the gain saturation is shown in Fig. 3. Depending on the light frequency ν, the change $\Delta n(U)$ may be both monotonous and nonmonotonous. Moreover, not only the absolute value, but also the sign of Δn may change. At the gain saturation, where $h\nu_{\rm exc} < \Delta F_0$, the value of ν may occur in the interval of absorption with the growing U at $\Delta F < h\nu$.

The inversion frequency, at which the sign of Δn changes, depends on the shape of the broadening form function, that may be used for experimental determination of the character of the broadening of spectral lines.

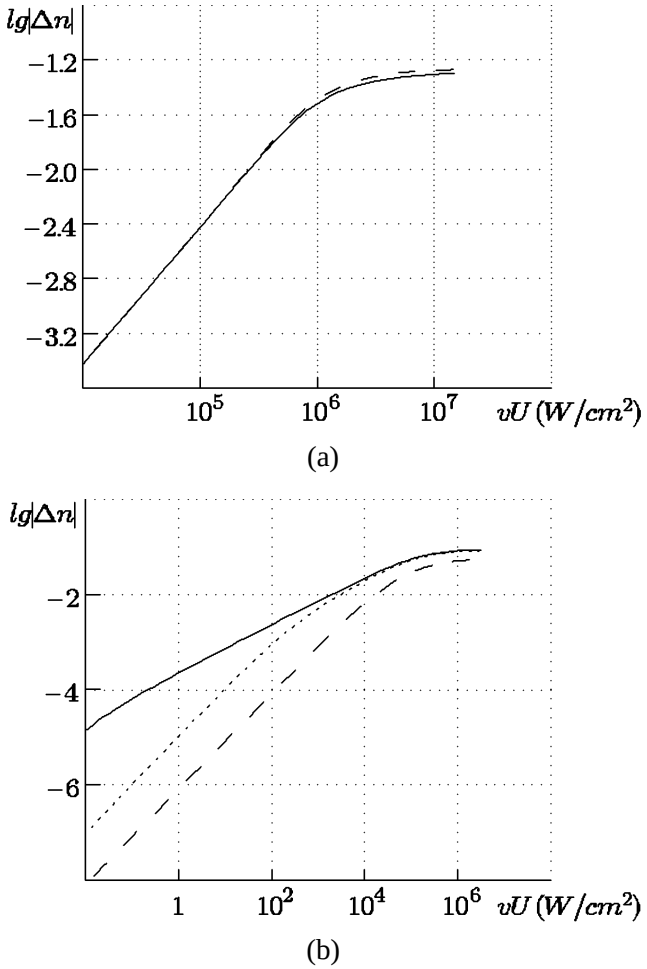

Fig. 4. The dependence of the change in the refractive index Δn on the density of radiation U (a) at the gain saturation ($\Delta F_0 = 1.67$ eV, $h\nu_{\rm exc} = 1.524$ eV) and (b) at the absorption saturation ($\Delta F_0 = 0$, $h\nu_{\rm exc} = 1.678$ eV) for the undoped QW (solid curve) and doped active region with the concentration of donors $N_{\rm d} = 2{\cdot}10^{17}$ cm^{-3} (dotted curve) and $2{\cdot}10^{18}$ cm^{-3} (dashed curve). $d =$ 5 nm, $h\nu = h\nu_{\rm 1h} = 1.521$ eV, TE mode.

As seen from Fig. 1, with growing U the frequency of inversion is shifted (with respect to $E_{\rm g}$) to the short-wavelength region at the gain saturation, while at the absorption saturation it shifts to the long-wavelength region [16].

The performed calculations and analysis demonstrate that the change in the refractive index Δn with the increase of the density of radiation U under conditions of the gain saturation follows almost a linear function and does not practically depend on the doping level, in contrast to the conditions of the absorption saturation (Fig. 4) [16]. For taking into account the influence of the nonlinear gain effects on the waveguide characteristics of laser heterostructures, the value of n_2/v of the order of $4{\cdot}10^{-5}$ cm^2/kW can be used.

References

[1] F. Stern, Dispersion of the index of refraction near the absorption edge of semiconductors, Phys. Rev. A **133** (6), 1653–1664 (1964).

[2] V.K. Kononenko and V.P. Gribkovskii, Saturation effect in semiconductor amplifiers of light and filters, Opt. Spectr. **29** (5), 975–984 (1970).

[3] P.G. Eliseev and A.P. Bogatov, Phenomena in semiconductor lasers related with nonlinear refraction and influence of current carriers on the refractive index, Proc. FIAN **166**, 15–51 (1986).

[4] A.N. Pikhtin and A.D. Yas'kov, Refraction of light in semiconductors, Sov. Phys. Semicond. **22** (6), 969–991 (1988).

[5] K.B. Kahen and J.P. Leburton, Exciton effects in the index of refraction of multiple quantum wells and superlattices, Appl. Phys. Lett. **49** (12), 734–736 (1986).

[6] S.H. Park, N. Peyghambarian, J.F. Morhange, R.A. Morgan, A.D. Jeffery, A. Chavez-Pirson, H.M. Gibbs, S.W. Koch, M. Derstine, A.C. Gossard, J.H. English, and W. Wiegmann, Comparison of the optical nonlinearities of bulk GaAs and GaAs/AlGaAs multiple quantum wells, Proc. SPIE **881**, 138–143 (1988).

[7] V.K. Kononenko, Nonlinear absorption in quantum-size heterostructures, Phys. Status Solidi B **150** (2), 695–698 (1988).

[8] V.K. Kononenko, Spectrum of non-linear refraction in quantum-well heterostructures, Laser Technique and Optoelectronics, (3–4), 75–79 (1993).

[9] D.V. Ushakov, V.K. Kononenko, and I.S. Manak, Nonlinear optical processes in doped semiconductor superlattices, J. Appl. Spectr. **68** (4), 656–662 (2001).

[10] V.K. Kononenko and S.V. Nalivko, Spectral characteristics of asymmetric quantum-well heterostructure laser sources, Proc. SPIE **2693**, 760–767 (1996).

[11] M. Asada, Intraband relaxation time in quantum-well lasers, IEEE J. Quantum Electron. **25** (9), 2019–2026 (1989).

[12] V.K. Kononenko, I.S. Manak, and E.R. Furunzhiev, Gain saturation in quantum-well heterostructures, J. Appl. Spectr. **64** (6), 813–817 (1997).

[13] V.K. Kononenko and I.S. Zakharova, Laser parameters of quantum-well heterostructures, Preprint ICTP No.IC/91/63 (Trieste, 1991).

[14] D.V. Karasev, V.K. Kononenko, I.S. Manak, and D.L. Kharevich, Polarization characteristics of laser quantum well heterostructures, Laser Technique and Optoelectronics, (3–4), 52–55 (1992).

[15] D.C. Hutchings, M. Sheik-Bahae, D.J. Hagan, and E.W. Van Stryland, Kramers–Krönig relations in nonlinear optics, Opt. Quantum Electron. **24** (1), 1–30 (1992).

[16] V.I. Tsvirko and V.K. Kononenko, Nonlinear refraction in quantum-well heterostructures, Bull. Belarus. Univ. Ser. 1, (1), 40–47 (2002).

NETIESINĖ REFRAKCIJA KVANTINĖSE DUOBĖSE, ESANT STIPRINIMO ĮSOTINIMUI

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Santrauka

Ištirtos refrakcijos kvantinių duobių heterolazerių aktyviajame sluoksnyje netiesinės ypatybės. Atsižvelgta į spektro išplitimo įtaką, rastas netiesinės refrakcijos spektras ir įvertintas heterosandarų netiesinio lūžio rodiklis GaAs–AlGaAs sistemoje.