deformation makes it possible to obtain more initial data, which increases the accuracy of identification of the elastic modulus.

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## COUPLED BEM AND FEM IN DYNAMIC ANALYSIS OF TANKS FILLED WITH A LIQUID

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Thin-walled shells are widely used in many industries including aerospace, civil, marine, petrochemical and nuclear engineering, power machine building, wind power engineering and transport. In many circumstances these shells are subjected not only to static loads but also to dynamic disturbances and filled with internal fluid. Usually they are filled with oil, flammable or toxic liquids. Such facilities are fuel tanks, liquid storage tanks, oil and propellant storage containers. The influences of both media on each other must not be neglected in stress-strength analysis of these structural elements. So the interaction between the sloshing liquid and the shell structure has been the challenging field of research in many engineering applications. In most cases, discrete techniques, such as the Finite Element Method (FEM) and the Boundary Element Method (BEM) have been employed and continuously further developed with respect to accuracy and efficiency. In fact, it did not take long until some researchers started to combine the FEM and the BEM in order to profit from their respective advantages by trying to evade their disadvantages. A detailed review on different numerical models for fluid-structure interaction can be found, e.g., in [1]. Several studies have been carried out in the different fields of sloshing liquids. Evaluation of the natural frequencies and corresponding mode shapes of liquid sloshing in a tank, linear and non-linear characters of the liquid flow, sloshing analysis in low and zero gravity, optimization and control of sloshing characteristics are some of researcher's favorite fields. Such research is needed to better understand the processes and help reduce the probability and aftermath of these tanks destruction due seismic actions or shockwaves that can lead to environmental catastrophe.

The dynamic analysis of shell structures is often performed by use the finite element programs. But such 3-D finite element analysis, including the contained fluid is complex and extremely time consuming. In [2-4] authors offer the approach based on using the boundary element method to the problem of natural vibrations of the fluid-filled elastic shells of revolution, as well as to the problem of natural liquid vibrations in the rigid vessels. The research findings are summarized in [5].

In this paper the coupled problem of free and forced vibrations of shell structures interacting with the fluid is under consideration. For its solution we use combination of reduced finite and boundary element methods. The analysis consists of several stages, each represents a separate task. The frequencies and modes of shell vibrations in a vacuum are defined by the first stage. Displacement vector, that is the solution of the hydrodynamic problem, is sought as a linear combination of the natural modes of shell vibrations in vacuum. Besides disturbing force, the shell is under the hydrodynamic pressure. The fluid is considered ideal, incompressible. Using the Cauchy-Lagrange integral value of this pressure can be expressed with velocity potential. For the velocity potential we have the boundary condition of impermeability connecting it with the normal component of displacement. The velocity potential is also found in the form of linear combination of boundary problem solutions of the Laplace equation for each vibration mode. The definition of these potentials is the second stage of the solution of the coupled hydroelasticity problem. Then we obtain the frequencies and free vibrations modes of a liquid in rigid shell under force of gravity. The latter two problems are solved using reduced BEM. Then we come to second order system of differential equations for forced vibrations of the shell partially filled with a liquid and solve it numerically using Runge-Kutta method.

To determine the frequencies and modes of free vibrations of the shell in a vacuum we use the finite element method. The motion equation of the shell in the absence of external disturbances is written on the basis of the Ostrogradskii-Hamilton principle. Displacement vector is represented in the form of Fourier expansion in the circumferential coordinate. Next, form the stiffness and mass matrix and reduce the dynamic problem for the shell in a vacuum to eigenvalue problem. In the second stage we apply potential theory and reduce the coupled boundary problem for the Laplace equation to a singular integral equation system for each mode. The solutions of these systems are also presented in the form of Fourier series in the circumferential coordinate. There is efficient numerical algorithm for solving such systems based on the application of the boundary element method [6–7]. Note that using combination of finite, boundary element methods and expansion into Fourier series gives great advantages in the analysis of complex structures.

In this paper the numerical procedure based on a coupling the finite element formulation and the boundary element method is developed for the forced vibration analysis of shells of revolution with an arbitrary meridian partially filled with the fluid. We obtained numerical results for dynamic problems for the cylindrical and conical shells interacting with a liquid both in the absence of the disturbing force (free vibrations) and with given external dynamic effects (forced vibrations). Also we obtain the frequencies and free vibrations modes of liquid in rigid shell under force of gravity. For cylindrical shell we investigate the influence of different filling levels on frequencies and modes of vibrations. And for conical shell the research was carried out concerned with varying cone slope and its influence on displacements.

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# АНАЛИЗ НАПРЯЖЕННО-ДЕФОРМИРОВАННОГО СОСТОЯНИЯ ГИБКОЙ СТЕНКИ В УПРУГОЙ СРЕДЕ С УЧЕТОМ КОНСТРУКТИВНОЙ И ФИЗИЧЕСКОЙ НЕЛИНЕЙНОСТЕЙ

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В работе выполнен подробный анализ современного состояния задачи и объекта исследования – гибкой стенки, как конструкции, широко используемой в условиях плотной городской застройки. В частности, показаны: особенности использования, конструкции и расчета шпунтовых ограждений. По условиям строительства и эксплуатации подпорные и ограждающие конструкции испытывают сложный характер нагружений и перемещений. Поэтому при проектировании необходимо учитывать совместную работу грунтового основания и самой конструкции.

Экспериментальные исследования работы подпорной стенки, позволяют сделать вывод, что характер действия нагрузки на подпорную стенку более соответствует предпосылкам теории упругости, чем теории предельного равновесия.

Анализ программных комплексов, имеющих в своей основе МКЭ, показал, что в них не реализуются расчеты, учитывающие одновременно два типа нелинейностей (физическую и геометрическую).

Широкое применение теория расчета тонкостенных элементов, взаимодействующих с деформируемым основанием, находит в строительстве. В рамках работы был выполнен обзор основных деформационных теорий грунта и его моделей, существующих методов расчета ограждающих конструкций.

К настоящему времени проблема выбора расчетной модели основания и метода расчета решается неоднозначно. По-видимому, дальнейшее развитие этого вопроса вряд ли приведет к созданию единой универсальной модели,