

$$a^2 m^2 \left(\frac{B_{11}^{(r)}}{B_{22}^{(r)}} (y^{(r)})^4 - \frac{2(B_{12}^{(r)} + 2B_{66}^{(r)})}{B_{22}^{(r)}} (y^{(r)})^2 + 1 \right) - \frac{B_{66}^{(r)}}{B_{221}^{(r)}} (\eta_m^{(r)})^2 = 0, r = 1, 2.$$

with positive real parts. $B_{ij}^{(r)}$, $r = 1, 2$, are coefficients of elasticity of composed shells

$$a^2 = k^2 \frac{h^2}{12}, (\eta_m^{(r)})^2 = \frac{\rho^{(r)} \omega^2}{m^2 k^2 B_{66}^{(r)}}, R^{-1} = k r_0 / 2, m = \overline{1, \infty}.$$

R is the radius of the directing circle, ω is the angular frequency, $\rho^{(r)}$, $r = 1, 2$, are the densities of the materials, $k = 2\pi/2$ for closed cylindrical shells and $k = \pi/s$ for non-closed shells, s is the length of directing circle, m is the wave number. From equation (1) it follows, that at $r_0^2 / m^2 \rightarrow 0$ the dispersion equation of the considered problems split into equations

$$G(\eta_m^{(1)}, \eta_m^{(2)}) = 0, L(\eta_m^{(1)}, \eta_m^{(2)}) = 0, K_3^{(1)}(\eta_m^{(1)}) = 0, K_3^{(2)}(\eta_m^{(2)}) = 0. \quad (2)$$

The first two equations from (2) are analogous to Stoneley dispersion equation for bending and plane vibrations composed of infinite plate and plate-strip [3]. Plane interfacial vibrations of composed cylindrical shell correspond to the roots of the third and forth equations of (2). This manifests the fact of using the equation of the corresponding classical theory of orthotropic cylindrical shells [1]. Numerical analysis shows the efficiency of the asymptotic formulas (1) and (2).

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ON THE APPLICATION OF THE I. VEKUA'S METHOD FOR THE GEOMETRICALLY NONLINEAR THEORY OF NON-SHALLOW SPHERICAL SHELLS

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I. Vekua constructed several versions of the refined linear theory of thin and

shallow shells, containing the regular processes by means of the method of reduction of 3-D problems of elasticity to 2-D ones [1].

In the present paper we consider non-shallow spherical shells [2]. The components of the deformation tensor have the following form:

$$e_{ij} = \frac{1}{2} \left(\vec{R}_j \partial_i \vec{u} + \vec{R}_i \partial_j \vec{u} + \partial^k \vec{u} \partial_k \vec{u} \right),$$

where \vec{R}_i are covariant basis vectors, \vec{u} is the displacement vector.

By means of I. Vekua method the systems of two-dimensional equations are obtained. Using the method of the small parameter, approximate solutions of these equations are constructed [3], [4]. The small parameter $\varepsilon = h/R$, where $2h$ is the thickness of the shell, R is the radius of the middle surface of the spherical shell. Some boundary value problems are solved for the approximation of order $N=1$.

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SOME PROBLEMS OF EQUILIBRIUM AND STABILITY OF NONLINEARLY ELASTIC CIRCULAR MEMBRANES

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Introduction. This paper presents a numerical-analytical algorithm for determining points of branching for circular membranes with arbitrary profile along the meridian. This class of structures includes a large number of corrugated membranes used as elastic elements in the devices of precision mechanics [1]. Spherical dome with a possible deviation from the ideal surface can be considered as a special case.

In the eighties of the last century, experiments conducted by S. Yamada [2], including high-precision measurements of the distribution of initial geometrical imperfections and vertical displacements in both subcritical and post-critical equilibrium states of a spherical dome showed the decisive influence of imperfections on the critical pressure and the shape of the stability loss. In this paper we show that by making