py. For anisotropic plates, the difference might be more pronounced. For the first-order theory, Ambartsumyan's and Huber's equations coincide.

Conclusion. The first-order consistent theories of Kirchhoff and Huber for isotropic and anisotropic plate materials, respectively, cannot be improved by adding in an unsystematic way single terms in the displacement ansatz or by assuming specific stress distributions in thickness directions. A consistent refined theory without a-priori assumptions results from the uniform-approximation technique in combination with the pseudo-reduction method.

References

1. Koneko T. *On Timoshenko's correction for shear in vibrating beams* // Journal of Physics D. – 1975. – Vol. 8. – P. 1927 – 1936.
2. Altenbach J., Altenbach H., Eremeyev V. A. *On generalized Cosserat-type theories of plates and shells*: *a short review and biography* // Archive of Applied Mechanics – 2010. – Vol. 80. – P. 72 – 92.
3. Schneider P., Kienzler R. *An algorithm for the automatization of pseudo reductions of PDE systems arising from the uniform-approximation technique* // Shell like structures: Non-classical theories and applications. Altenbach H., Eremeyev V. A. (eds.) Advanced Structured Materials, Springer: Berlin – 2011. – Vol. 15. – P. 377 – 390.
4. Ambartsumayan S. A. *Theory of anisotropic plates*. Progress in Material Sciences II. Stamford (Conn.): Technicon. – 1970.
5. Kienzler R. *On consistent plate theories*// Archive of Applied Mechanics. – 2002.–P. 225 – 247.
6. Kienzler R. *On consistent second-order plate theories* // Theories of plates and shells. Critical review and new applications. Kienzler R., Altenbach H., Ott, I. (eds.). Lecture Notes in Applied and Computational Mechanics, Springer: Berlin –2004. – Vol. 16. – P. 85 – 96.
7. Schneider P., Kienzler R., Böhm M. *Modeling ofconsistentsecond-order platetheories-foranisotropicmaterials* // Zeitschrift für Angewandte Mathematik und Mechanik. – 2013. – DOI: 10.1002/zamm.201100033.
8. Huber M. T. *Some applications of the theory of bending of orthotropic plates*// Zeitschrift für Angewandte Mathematik und Mechanik. – 1926. – Vol. 6. – P. 228 – 231.

THREE-DIMENSIONAL EXACT ANALYSIS OF FUNCTIONALLY GRADED AND LAMINATED PIEZOELECTRIC PLATES

Kulikov G. M., Plotnikova S. V.
Tambov State Technical University, Department of Applied Mathematics and Mechanics
Sovetskaya Street, 106, 392000 Tambov, Russia
gmkulikov@mail.ru, plotnikovasvetlana62@gmail.com

Three-dimensional (3D) static analysis of functionally graded and laminated piezoelectric plates has received considerable attention during past twenty years. There are at least three approaches to 3D exact solutions of electroelasticity for functionally graded and laminated piezoelectric plates, namely, the Pagano approach, the state space approach and the asymptotic approach (see, e.g. [1]). These approaches were applied efficiently to 3D exact solutions for piezoelectric plates in many contributions. However, the 3D exact analysis for laminated piezoelectric plates of general lay-up configurations can not be found in the current literature; only developments for cross-ply and angle-ply composite structures in the framework of 3D anisotropic elasticity are available.

To solve such a problem, we invoke a new efficient method of sampling surfaces (SaS) proposed recently by the authors [2–4] for homogeneous and laminated plates and shells. As SaS $\Omega^{(n)1}, \Omega^{(n)2}, ..., \Omega^{(n)I_n}$, we choose outer surfaces and any inner surfaces inside the *n*th layer and introduce displacement vectors $\mathbf{u}^{(n)1}, \mathbf{u}^{(n)2}, ..., \mathbf{u}^{(n)I_n}$ of these surfaces as basic plate variables, where I_n is the total number of SaS chosen for each layer ($I_n \geq 3$). Such choice of displacements with the consequent use of Lagrange polynomials of degree $I_n - 1$ in the thickness direction for each layer permits the representation of governing equations of the piezoelectric laminated plate formulation in a very compact form. It is necessary to note that the term SaS should not be confused with such terms as fictitious interfaces or virtual interfaces, which are extensively used in layer-wise theories. The main difference consists in the lack of possibility to employ the polynomials of high degree in the thickness direction because in conventional layer-wise theories only the third and fourth order polynomial interpolations are admissible [5, 6]. This restricts the use of the fictitious/virtual interfaces technique for derivation of 3D exact elasticity solutions. On the contrary, the SaS method permits the use of polynomials of high degree. This fact gives in turn the opportunity to derive 3D exact solutions for laminated composite plates and shells with a prescribed accuracy employing a sufficient number of not equally spaced SaS.

It is important to mention that the developed approach with equally spaced SaS does not work properly with Lagrange polynomials of high degree because the Runge's phenomenon can occur, which yields the wild oscillation at the edges of the interval when the user deals with any specific functions. If the number of equally spaced nodes is increased then the oscillations become even larger. Fortunately, the use of Chebyshev polynomial nodes can help to improve significantly the behavior of Lagrange polynomials of high degree for which the error will go to zero as $I_n \to \infty$.

References

1. Wu C. P., Chiu K. H., Wang Y. M. *A review on the three-dimensional analytical approaches of multilayered and functionally graded piezoelectric plates and shells* // Computers, Materials & Continua. – 2008. – Vol. 8. – P. 93 – 132.
2. Kulikov G. M., Plotnikova S. V. *On the use of sampling surfaces method for solution of 3D elasticity problems for thick shells* // ZAMM - Journal of Applied Mathematics and Mechanics. – 2012. – V. 92. – P. 910 – 920.
3. Kulikov G. M., Plotnikova S. V. *Exact 3D stress analysis of laminated composite plates by sampling surfaces method* // Composite Structures. – 2012. – Vol. 94. – P. 3654 – 3663.
4. Kulikov G. M., Plotnikova S. V. *Advanced formulation for laminated composite shells: 3D stress analysis and rigid-body motions* //Composite Structures. – 2013. – Vol. 95. – P. 236 – 246.

5. Carrera E. *Theories and finite elements for multilayered plates and shells: a unified compact formulation with numerical assessment and benchmarking* // Archives of Computational Methods in Engineering. – 2003. – Vol. 10. – P. 215 – 296.
6. Carrera E., Brischetto S., Nali P. *Plates and Shells for Smart Structures: Classical and Advanced Theories for Modeling and Analysis.* John Wiley & Sons Ltd. – 2011.

THIN COMPOSITE SHELLS LIKE «SANDWICH» CONTAINING MAGNETORHEOLOGICAL ELASTOMERS: VIBRATIONS AND THEIR SUPPRESSION

Mikhasev G. I.
Belarusian State University, 4 Nezavisimosty Ave., 220030, Minsk, Belarus
mikhasev@bsu.by

The vibroprotection of thin-walled structures experiencing an external vibrational load is a subject of great practical interest for mechanical engineers which design and model similar structures. The appearance of the group of new composite materials with active and adaptive properties, called smart materials, opens new possibilities for solving these problems [1]. Some of these composites are magnetorheological (MR) ones and, particularly, magnetorheological elastomers (MRE). They belong to the group of active materials which physical properties such as viscosity and shear modulus can vary when subjected to different magnetic field levels [2].

Laminated cylindrical shells and beams like «sandwich» formed by embedding MRE in between elastic layers are the subject of this study. Similar composite structures are very popular in aerospace and in many other industries due to their light weight and high- energy absorption properties of the MR layers.

The MRE are magnetizable particles molded in either rubbery polymers or deformed inorganic polymer matrices. The optimum weight/density ratio of magnetic particles, carrier viscous liquid and polymer determine shear modulus, viscosity and response time being the integral characteristics of a smart material. Physical properties of the MR layers are assumed to be functions of the magnetic field induction [3]. Because the influence of the magnetic field on all areas of the MR lamina is different [4], it is assumed inhomogeneity of physical properties of MRE.

In general case, the shell structure may be non-circular and not closed in the circumferential direction (cylindrical panel). A system of differential equations with complex variable coefficients depending upon the magnetic field [4], and based on both the assumptions of the generalized kinematic hypothesis for the whole «sandwich» [5] and experimental data for MRE [6], is utilized as governing one.

To analyze damping capabilities of adaptive materials, free vibrations of a three-layered beam [6] and circular cylinder containing interlayer MRE are studied at different levels of the magnetic field. Then the case when applied magnetic field results in nonuniformity of MRE is considered. Using the asymptotic approach [7], eigenmodes of free vibrations of the laminated noncircular cylindrical shell with variable physical characteristics of MRE are constructed in the form of functions decaying far from the weakest plot on the shell structure. It has been shown that applying constant magnetic field may result in strong localization of eigenmodes cor-