References

1. Gurtin M. E., Murdoch A. I. *A continuum theory of elastic material surfaces* // Arch. Ration. Mech. And Anal. – 1975. – Vol. 57, No. 4. – P. 291 – 323.
2. Murdoch A. I. *Some fundamental aspects of surface modeling*. J. Elast. – 2005. – Vol. 80. – P. 33 – 50.
3. Podstrigach Ya. S., Povstenko Yu. Z. *Introduction to the mechanics of surface phenomena*. – 1985, Kiev.
4. Duan H. L., Wang J., Karihaloo B. L. *Theory of elasticity at the nanoscale* // Advances in Applied Mechanics. – 2008, No. 42. – P. 1 – 68.
5. Altenbach H., Eremeyev V. A., Morozov N. F. *Linear theory of shells taking into account surface stresses* // Dokl. Phys. – 2009. – Vol. 54 – .P. 531 – 535.
6. Goldstein R. V., Gorodtsov V. A., Ustinov K. V. *Effect of residual stress and surface elasticity on deformation of nanometer spherical inclusions in an elastic matrix* // Phys. Mesomechanics. – 2010. – Vol. 13. – P. 318 – 328.
7. Eremeev V. A., Morozov N. F. *The effective stiffness of a nanoporous rod* // Dokl. Phys. – 2010. – Vol. 55, No. 6. – P. 279 – 2820.
8. Grekov M. A., Morozov N. F., Yazovskaya A. A. *A circular hole in an elastic material at the nanoscale. Topical Problems in Solids and Fluid Mechanics*. – New Delhi: Elite Publ. House. – 2011. – P. 12 – 18.
9. Timoshenko S. P., Woynovsky-Krieger S. *Theory of plates and shells*. – M.: Nauka. – 1966.
10. Volmir A. S. *Stability of Deformable Systems*. – M.: Nauka. – 1967.
11. Guz A. N., Dyshel M. Sh., Kuliev G. G., Milovanova O. B. *Destruction and stability of thin bodies with cracks*. – Kiev. – 1981.
12. Bochkarev A. O., Dal' Yu. M. *Local stability of notched elastic plates* // Soviet Physics Doklady. – 1989. – Vol. 34. – P. 854.
13. Tian L, Rajapakse R. K. D. *Analytical solution for size-dependent elastic field of a nanoscale circular inhomogeneity* // Trans. ASME. J. Appl. Mech. – 2007. – Vol. 74 (5). –P. 568 – 574.
14. Musheshvili N. I. *Some basic problems of the mathematical theory of elasticity*. P. Noor-hoff, Groningen, The Netherlands. – 1963.
15. Morozov N. F., Petrov Yu. V. *Dynamics of Fracture*. Springer, 2000.
16. Ronald E., Miller and Vijay B Shenoy. *Size-dependent elastic properties of nanosized structural elements* // Nanotechnology. – 2000. – Vol. 11. – P. 139 – 147.

INTERNAL GEOMETRY OF NANOSHELLS AND WAVES PROPAGATION

Miklashevich I. A. [†], **Erofeyev V. I.** [*]

[†] Belarusian National Technical University and Belarusian State University,
miklashevich@yahoo.com

[*] Mechanical Engineering Research Institute of the Russian Academy of Sciences,
NishnyNovgorod, Russia
erf04@sinn.ru

Introduction. Carbon nanotubes are the real objects which help to limit the choice of representative volume. As regard nanotubes, the unit volume corresponds to the nanotube itself and the ideal continuum, which is presented as a set of inter-

acting carbon rings, prevents from evident entrance of defects. Deviation from ideality (chirality of nanotubes) is one of the basic characteristics of the material. One may get an impression that a nanotube in itself is one single macroscopic defect. In this connection, definition of metric properties of a nanotube's "initial" state, namely "nanoshells" makes undoubtful interest.

Formalism description. Let us consider a possibility to describe carbon nanotubes by using the differential geometry of the spaces formalism. It is known that chirality indices of nanotubes i_1, i_2 define the vector: $\mathbf{C} = i_1\mathbf{T}_1 + i_2\mathbf{T}_2$, where $\mathbf{T}_1, \mathbf{T}_2$ are vectors of elementary translations. These indices define two screw translations with angles and steps:

$$\delta\varphi_i = 2\pi\frac{\mathbf{T}_i \cdot \mathbf{C}}{\mathbf{C}^2},\ \Delta z_i = \mathbf{T}_i \cdot \mathbf{e}_i,\ i = 1,2,2$$

where $\mathbf{e}_i$ is the unit vector, directed along the band generator.

However, according the Halphen theorem, the composite of two screw translations is the screw translation, the parameters of which φ, z, ϕ can be defined in an elementary way [1]. Let the ς be an elementary vector, parallel to the side of the carbon ring in the initial state.

According to the traditional procedure of encirclement in the space of imperfect (non-ideal) crystals (which is geometrically equivalent to the space of absolute parallelism [2]), the nullity vector of the end and start of a looped curve is equal to:

$$\Delta u^k = -\Gamma^k_{[\mu\nu]}\, d_2 x^{[\mu}\, d_1 x^{\nu]} = 2S^k_{\mu\nu} f^{\mu\nu}, \tag{1}$$

where $f^{\mu\nu}$ is the bivector of the 2-D area related to the infinitesimal loop and $S^k_{\mu\nu}$ is the torsion tensor, $\Gamma^k_{[\mu\nu]}$ is the connectivity coefficients [3]. The vector rotation angle at encirclement is defined by the expression:

$$\Delta\varphi_\lambda = d_2 d_1 \mathbf{v}_\lambda - d_1 d_2 \mathbf{v}_\lambda = R^k_{\lambda\mu\nu} f^{\mu\nu} \tag{2}$$

where $R^k_{\lambda\mu\nu}$ is the first curvature tensor [4]

$$R^k_{\lambda\mu\nu} = \frac{\delta F^i_{jk}}{\delta x^i} - \frac{\delta F^i_{jl}}{\delta x^k} + F^m_{jk}F^i_{ml} - F^m_{jl}F^i_{mk} + C^i_{jm}F^m_{kl}. \tag{3}$$

In (3) $F^i_{jk}(x, y)$ and $C^i_{jk}(x, y)$ are coefficients of this connectivity [4]. Taking into account (2), (1) the deformation, connected with stretching and turn of the continuum (carbon monolayer), are presented as:

$$\delta\varsigma^\kappa = -\frac{1}{2}\left(R^\kappa_{\mu\nu\lambda}\varsigma^\lambda + 2C^\kappa_{\mu\nu}\right)df^{\mu\nu}, \tag{4}$$

where $R^k_{\lambda\mu\nu}$ is the curvature tensor, $C^k_{\mu\nu}$ is the torsion tensor, $df^{\mu\nu}$ is the bivector of the 2-D area related to the infinitesimal loop.

The torsion tensor can be easily related with the rotation vector, and the elastic deformation of the continuum is the deviation measure of the internal metric away from the external one. In this case, the compatibility conditions of Saint-Venant pass over into the condition of equality to zero of the Riemann – Christoffel tensor components. Outgoing from the elastic energy, we can define all the mechanical properties of nanotubes.

Coiled carbon nanotubes. Coiled carbon nanotubes are a subject of additional interest. A nanotube coiling is possible by two principally different methods: firstly, by coiling of a tube onto imaginary (virtual) cylinder having some definite radius *R*. In this case, the deformation is defined according to (4).

Secondly, by twisting a rectilinear tube around its own axis until appearance of instability, and deformation of the rectilinear generatix of the cylindrical shell into a spiral structure. In this case, the general deformation of the tube continuum with account of the expression of relative curvature [3] will be defined as:

$$\delta\varsigma^{\kappa} = -\frac{1}{2}\left(R^{...\kappa}_{\mu\nu\lambda}\varsigma^{\lambda} + \tilde{K}^{...\kappa}_{\nu\mu\lambda}\varsigma^{\lambda} + 2S^{...\kappa}_{\mu\nu} \right) df^{\mu\nu}.$$

Since in the second case the total deformation is much higher, the elasticity modulus of coiled nanotubes of the second type should have a compression-stretching asymmetry, and by stretching it should essentially exceed the elasticity modulus of coiled nanotubes of the first type.

Waves in nanotubes. According to general methodology all kinds of the waves in continua with microstructure can be obtained as solution of the variational problem for density of the Lagrange function [5]. It means that equation for the geodesic line give us equation for the wave trajectory.

Let the geodesic line is set in the form of $x^i = x^i(t), a \le t \le b$. Let the parallel displaced tangent vector is ξ^i. Outgoing from the fact that collinear vectors are parallel, we may pass over to the canonical parameter along curve τ. For this parameter:

$$\frac{dx^i}{d\tau} = -\xi^i.$$

Making use of the definition of the geodesic line and the definition of parallel displacement follow from (1), we have:

$$d\frac{dx^k}{d\tau} = -\Gamma^k_{ij}\frac{dx^j}{d\tau}dx^i.$$

By dividing the right- and left-hand parts by $d\tau$, we have a geodesic line equation in the form referred to the canonical parameter:

$$\frac{d^2x^k}{d\tau^2} = -\Gamma^k_{ij}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau}. \qquad (5)$$

Formula (5) may be considered as a system of second-order non-linear differ-

ential equations, the integral lines of which are geodesics.

References

1. Whittaker E. T. *A threatise of the analitical dynamics of particles and rigid bodies* // Cambridge: At the University Press. – 1927.
2. Onami M., Ivasimadzu S., Genka K., Siodzava K., Tanaka K. *Vvedenije v mikromechaniku* // Moskwa: Metallurgija, – 1987. – 280c. Russian translation from Japan.
3. Miklashevich I. A. *Micromechanics of fracture in generalized spaces* // Amsterdam-Boston: Academic Press. – 2008. – 296 p.
4. Miron R. *Introduction to the theory of Finsler space* // Theproceeedings of the national seminar on Finsler spaces, Februaryю – 1980. Timisoara, 1981. P. 131 – 183.
5. Erofeyev V. I. *Wave processes in solids with microstructure* // New Jersey-London-Singapore-Hong Kong-Bangalore-Taipei: World Scientific Publishing. – 2003. – 256 p.

ON STABILITY OF INHOMOGENEOUS ELASTIC TUBES OF MICROPOLAR MATERIALS

Sheydakov D. N., Bocharova O. V.
Southern Scientific Center of Russian Academy of Sciences
Chekhov Ave., 41, 344006 Rostov-on-Don, Russia
sheidakov@mail.ru

The problem of equilibrium stability for deformable bodies is of major importance both from theoretical and practical points of view, because the exhaustion of load-carrying capability and collapse of buildings and engineering structures quite often occurs due to buckling under external loads. In the case of elastic medium, the stability theory is extensively developed for classic non-polar materials. There is large number of studies on stability both for thin and thin-walled bodies in the form of rods, plates and shells, and for massive (three-dimensional) bodies. However, due to the increasing number of new structural materials, the problem of stability analysis for bodies with a microstructure becomes relevant. One example of such materials is a porous material. Engineering structures made of porous materials, especially metal and polymer foams, have different applications in the last decades [1–4]. The foams are cellular structures consisting of a solid metal (for example aluminium, steel, copper, etc.), or polymer (polyurethane, polyisocyanurate, polystyrene, etc.) and containing a large volume fraction of gas-filled pores. There are two types of foams. One is the closed-cell foam, while the second one is the open-cell foam. The defining characteristic of metal and polymer foams are the very high porosity: typically, well over 80%, 90% and even 98% of the volume consists of void spaces.

Constructions made of porous materials are widely used in modern industries with airspace or automotive applications among others. The reason for this is the advantages of such materials: better density-stiffness ratios in comparison with classical structural materials, the possibility to absorb energy, etc. As a rule, these constructions have a functionally graded structure. For example, the porous core is quite often covered by hard and stiff shell, which can be necessary for corrosion or thermal protection, and optimization of mechanical properties in the process of loading.

The present research is dedicated to the stability analysis of nonlinearly elastic