NUMERICAL EXPERIMENT TO PROGRAM AND FEEDBACK CONTROLS IN OBSERVATION PROBLEM.

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Abstract: In our paper we consider the previous [1], [2] models. The problems of observation are formulated for similar models [3]. Those problems are closely adjoined to pattern recognition. In paper [1] we describe an algorithm to construct a numerical optimal program and feedback controls in pulse functions class, in paper [2] - in measurable functions class consequently. Now we present results of numerical experience in MatLab 6 to all algorithms [1], [2] that constructs program and feedback optimal controls in a class of pulse and measurable functions. We make some conclusion about process of numerical solution.

Problem formulation

We consider the following parabolic system under uncertain measurable function $\omega(t), t\in T=[t_0, t^*]$ with no probabilistic information available and boundary controls $u(t), t\in T$:

$$\begin{cases} \dfrac{\partial\varphi(t,x)}{\partial t} = L_x\,\varphi(t,x) + \omega(t), & x\in(x_0, x^*),\ t\in(t_0,t^*); \\ \varphi\,(t_0,x) = 0; & x\in[x_0, x^*] \\ A\cdot\dfrac{\partial\varphi}{\partial x}(t,x^*) = \varphi\,(t,x_0) = u\,(t); & t\in(t_0,t^*]; \end{cases} \tag{1}$$

Where $L_x\varphi(t,x) = \dfrac{\partial}{\partial x}(A\cdot\dfrac{\partial\varphi}{\partial x}) + B\cdot\dfrac{\partial\varphi}{\partial x} + C\cdot\varphi,\ x\in(x_0,x^*)$ is *PDE* operator and its real coefficients $A(t,x), B(t,x), C(t,x)$ are functions under restrictions [4], that provides an existence of system solutions in generalized sense. The components of *PDE* system (1) – system state $\varphi(t,x)$, $(t,x)\in\Omega=[x_0, x^*]\times[t_0,t^*]$, boundary controls $u(t), t\in T$, perturbation function $\omega(t), t\in T$ – are also constrained:

$$b_* \le \varphi\,(t,x^*) \le b^*;\ (t,x)\in\Omega \tag{2}$$

$$d_* \le u(t) \le d^*;\ t\in T; \tag{3}$$

$$\omega_* \le \omega(t) \le \omega^*;\ t\in T; \tag{4}$$

So, the following feedback optimal control problem is considered in initial formulation *(P)*:

$$(P)\ \text{maximize}\ J(u) = \int_{t_0}^{t^*}\big(u\,(\varphi\,(t,x^*))\big)dt = \int_{t_0}^{t^*} u\,(t)\,dt$$

subject to (1) – (4).

Following our papers [1, 2] instead of constraints (3), that are too difficult to numerical realization, we firstly consider constraints

$$\varphi(\tilde{t}_k, x^*) = b_k;\ \tilde{t}_k \in (t_0, t^*);\ k \in K = \{k_1, k_2, ..., k'\} \quad (5)$$

to fixed points $\tilde{t}_k \in (t_0, t^*)$; $k \in K$. So, instead of problem *(P)*, now we consider the following feedback optimal control problem *(P1)*:

$$(P1) \text{ maximize } J(u) = \int_{t_0}^{t^*} u(t)\,dt$$

subject to (1), (3) - (5).

Note, that we constructed [1], [2] a numerical algorithms to program and feedback controls, which has been developed initially to ODE [3] and then generalized by us to PDE [1, 2, 5].

We present a of numerical experiment results of solving problem *(P1)* in pulse functions class (using discretization the initial PDE (1)). We also present a of numerical experiment results of solving problem *(P1)* in measurable functions class (without of digitization the initial PDE (1)). The suggested methods [1], [2] was programmed in MatLab 6.0 language. Calculations were executed by using computer P III - 500. To obtain the numerical solution to the differential system (2), (4) an iterative process to the four points scheme for nonzero elements was used.

Numerical experiences of contraction a program optimal control in pulse and measurable function class.

Below we publish two tables that contain the results of numerical experiment for two methods. The first table contains a statistics for a method of construction of optimal program controls in pulse function class (discretization) [1]. The second table contains a statistics for a method of construction of optimal program controls in measurable function class [2] (without discretization).

h	dim	k	n	ΔJ %	Dual method with long step			Direct method with long step		
					int	t_{int}, %	t	int	t_{int}, %	T
0.5	507	3	20	24	7	71	0.84	7	74.1	0.89
0.2	1824	3	50	25.7	13	74.8	2.63	19	80.6	2.70
0.05	9525	3	200	29	35	95	46.52	23	94.9	46.55
0.02	35187	3	500	30	67	97	690.3	69	98	695.7

Table 1 Construction of program optimal controls in pulse function class.

h	k	n	ΔJ_c, %	I	t	t_N	t_N, %
0.5	3	20	3	1	0.84	0.05	6
0.2	3	50	2.3	1	2.63	0.059	2.2
0.05	3	200	3.1	1	46.52	0.07	0.15
0.02	3	500	2.9	1	690.3	0.09	0.01

Table 2 Construction of program optimal controls in measurable function class

Here: h – a step of time discretization, dim – dimension of matrix to integrate conjugate differential systems, k – a number of check point (a number of problem state restrictions), n – a number of time steps, ΔJ - functional increment (to functional initial value), int – a number of conjugate differential systems integration, t_{int} – a common time of conjugate differential systems integration (in percents to common solution time of optimization problem), t – a common solution time of optimization problem (in seconds), I – a number of Newton methods iterations to solve a nonlinear equations system, t_N – a solution time of Newton methods, ΔJ_c – functional increase (in percents to functional optimal value in a case of discretization).

Numerical experiences of construction a feedback optimal control in pulse and measurable function class.

Below we publish two tables that contain the results of numerical experiment for two methods. The first table contains a statistics for a method of construction of optimal feedback controls in pulse function class (discretization) [1]. The second table contains a statistics for a method of construction of optimal feedback controls in measurable function class [2] (without discretization). Limits of disturbances function $\omega(t)$ do not more than two percent to maximum control value $u(t)$.

h	dim	k	n	ΔJ %	Dual method with long step			Direct method with long step		
					t_{max}	int	t_{int}, %	t_{max}	Int	t_{int}, %
0.5	507	3	20	23	0.9	4.7	77	0.91	5.1	74
0.2	1824	3	50	26.7	2.74	3.3	76.8	2.71	11	86
0.05	9525	3	200	27	48.53	20.1	90.6	48.64	15.3	95.8

Table 3 Construction of feedback optimal controls in pulse function class

h	k	n	ΔJ_c, %	I_m	$t_{c\,m}$	$t_{N\,m}$	$t_{N\,m}$, %
0.5	3	20	1.2	1	0.8	0.01	1.25
0.2	3	50	1.3	1	1.1	0.012	1
0.05	3	200	1.19	1	15	0.015	0.1

Table 4 Construction of feedback optimal controls in measurable function class

Here: h – a step of time discretization, dim – dimension of matrix to integrate conjugate differential systems, k – a number of check point (a number of problem state restrictions), n – a number of time steps, ΔJ - a functional increment (to functional initial value), t_{max} – a maximum time to construct feedback control during one step, int – a middle number of conjugate differential systems integration, t_{im} – a common time of conjugate differential systems integration (in percents to common solution time of optimization problem), ΔJ_c - functional increase (in percents to functional optimal value in a case of discretization), I_m – a middle number of Newton methods iterations to solve a nonlinear equations system, $t_{c\,m}$ – a middle time to construct feedback control, $t_{N\,m}$ – a middle solutions time of Newton method.

Some remarks on results of numerical experience.

Notice the following interesting aspects during optimal control problem calculation process. Almost all of that aspects are similar on marked in our paper [5] effects.

Remark 1. If we investigate an optimal control problem in a class of pulse functions we can see, that an increment of control problem quality criteria ΔJ is not large in comparison with initial value.

Remark 2. Studying a problem in a class of measurable functions (without discretization) does not give an essential increment of quality criteria in comparison with a class of pulse functions (without discretization). Thus complexity of optimal control problem qualitatively grows.

Remark 3. The statistic data presented in the table give us a reason to make a conclusion that the main resources of computer during solving process are used for numerical integration of differential systems solution.

Remark 4. A time for integration conjugate differential system very quickly grow with increase of problems dimension. Therefore for the large dimensions problem the process of optimal feedback controls construction in a mode of real time is difficult.

References

[1] A.V. Borzenkov, O.L. Konovalov. "Program and feedback numerical controls to parabolic optimization system with uncertain disturbances". Proceedings of International Conference of Modeling and Simulation, Spain, Santiago de Compostella, (1999), 90-100.

[2] A.V. Borzenkov, O.L. Konovalov. "Numerical Feedback Optimal Controls to Parabolic Optimization System with Uncertain Disturbances". Proceedings of International Conference of Modeling and Simulation, Spain, (2002), 82-90.

[3] R. Gabasov, F.M. Kirillova, S.V. Prichepova. Feedback optimal controls. Springer-Verlag. Berlin, Heidelberg, New York, London, Paris, Tokyo, Hong Kong. (1995).

[4] Mihailov V.P. Differential equations in partil derivatives (In Russian). M. Nauka. (1983), 424 p.

[5] A.V. Borzenkov, O.L. Konovalov. "Numerical results of the program solution for the elliptic optimal control problem". Proceedings of International Conference on Systems and Signals in Intelligent Technologies, Minsk, (1998), 31-36.